Pola

Kwant przewodnictwa

$$G = \frac{dI}{dU} = \frac{2e^2}{h}\int_{E_L}^{\infty} \frac{\partial f(E,\mu)}{\partial E} T(E) dE \approx \frac{2e^2}{h} T(\mu) = 2G_0' T(\mu) = G_0 T(\mu)$$

$$\frac{e^2}{h} = 38{,}7\ \mu S$$

To są różne definicje G_0 - częściej jest z 2:
$$\frac{2e^2}{h} = 77{,}4\ \mu S$$

Wzór Landauera (*Landauer formula*) – gdy mamy do czynienia z wieloma kanałami przewodnictwa

$$G = G_0 \sum_n T_n(\mu) \approx N G_0'$$

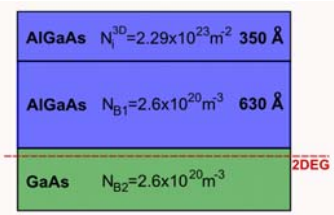
Trzeba uważać na definicję G_0!

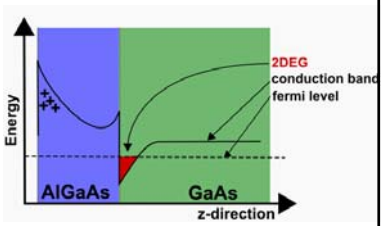
N to suma różnych „pełnych" kanałów przewodnictwa
np. dwa różne spiny dają $N = 2$

Dla $T = const$ dostajemy prawo Ohma.

Studnia trójkątna

Metoda przybliżona WKB (Wentzel – Krammers – Brillouin) – dla potencjału *wolnozmiennego*



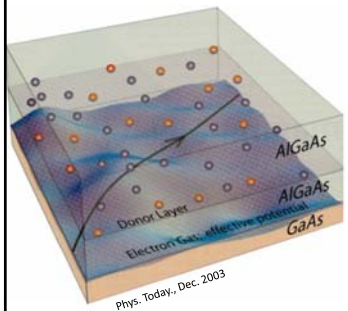


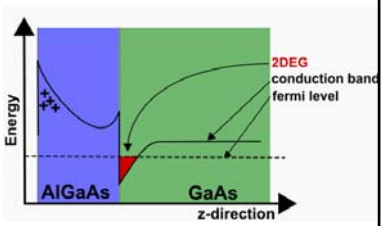
http://www.phys.unsw.edu.au/QED/research/2D_scattering.htm

$$E_n = \left[\frac{3}{2}\pi\left(n - \frac{1}{4}\right)\right]^{2/3}\left[\frac{(eF\hbar)^2}{2m}\right]^{1/3}$$

Studnia trójkątna

Metoda przybliżona WKB (Wentzel – Krammers – Brillouin) – dla potencjału *wolnozmiennego*





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$$E_n = \left[\frac{3}{2}\pi\left(n - \frac{1}{4}\right)\right]^{2/3}\left[\frac{(eF\hbar)^2}{2m}\right]^{1/3}$$

Kwant przewodnictwa

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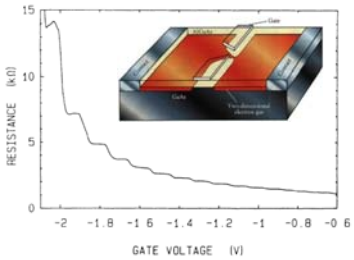


FIG. 1. Point-contact resistance as a function of gate voltage at 0.6 K. Inset: Point-contact layout.

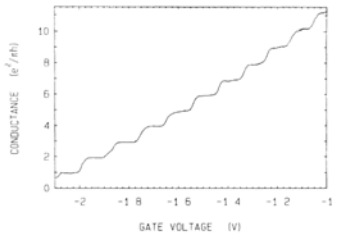
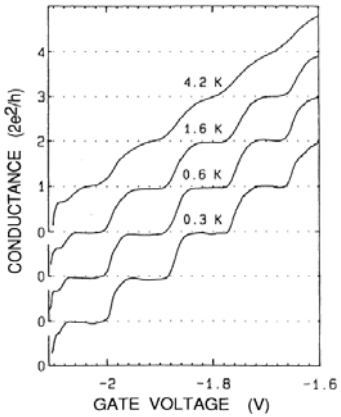


FIG. 2. Point-contact conductance as a function of gate voltage, obtained from the data of Fig. 1 after subtraction of the lead resistance. The conductance shows plateaus at multiples of $e^2/\pi\hbar$.

B. J. van Wees et al. *Quantized conductance of point contacts in a two-dimensional electron gas* Phys. Rev. Lett. **60**, 848–850 (1988)

Kwant przewodnictwa

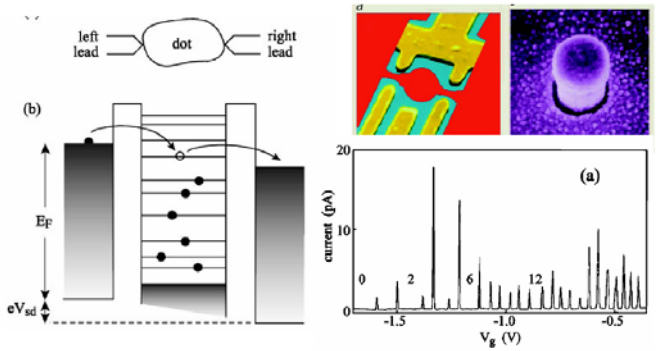
$$G = \frac{2e^2}{h} T(\mu) = G_0 T(\mu)$$



B. J. van Wees et al. *Quantum ballistic and adiabatic electron transport studied with quantum point contacts* Phys. Rev. B 43, 12431–12453 (1991)

FIG. 6. Breakdown of the conductance quantization due to temperature averaging. The curves have been offset for clarity.

Blokada Kulombowska



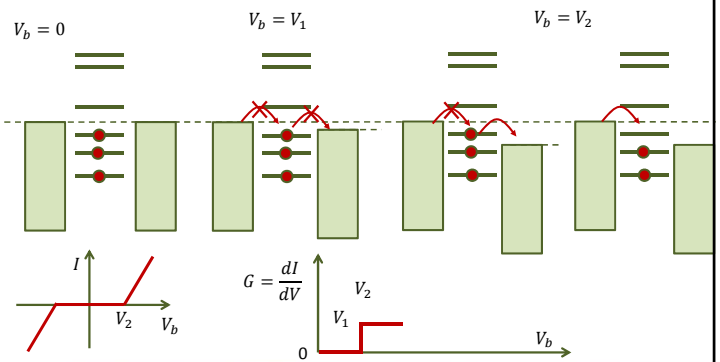
Coulomb Blockade in Quantum Dots: "dot spectroscopy"

http://sces.phys.utk.edu/~dagotto/condensed/Lectures_2008/UTK_Lecture1_March08.pdf

Luis Dias – UT/ORNL

Blokada Kulombowska

Kropka zachowuje się jak mały kondensator o energii $E_c \sim \frac{1}{2}\frac{e^2}{C}$



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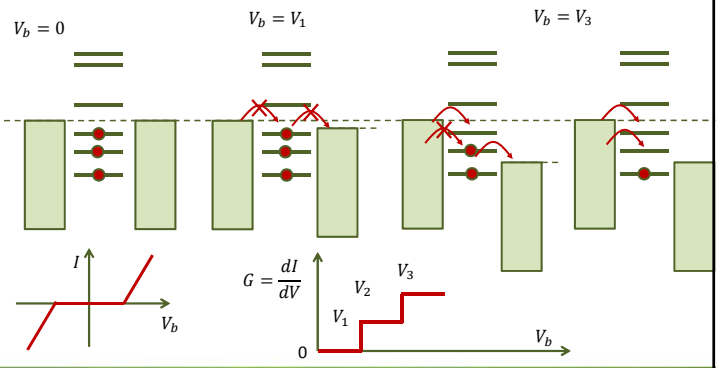

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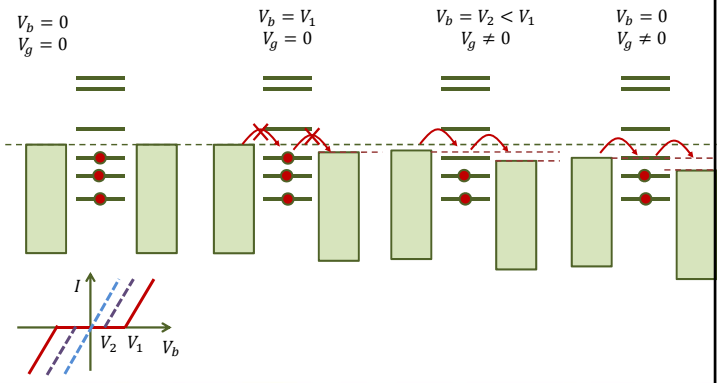
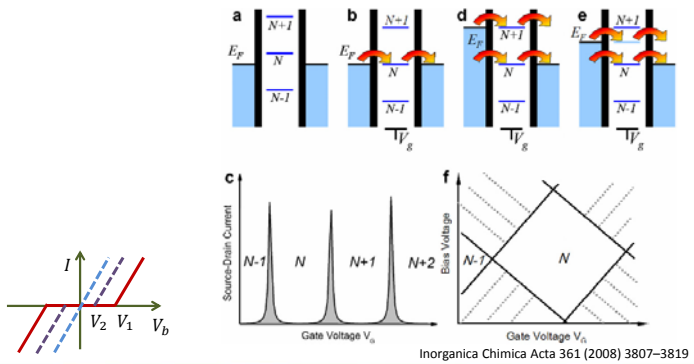


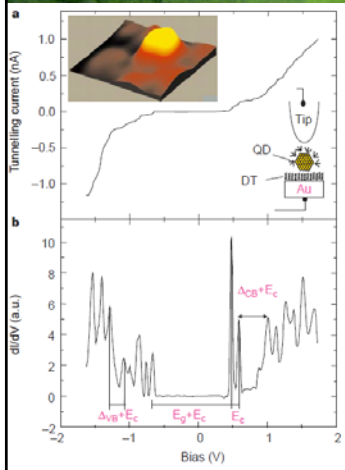
Blokada Kulombowska

Kropka zachowuje się jak mały kondensator o energii $E_c \sim \frac{1}{2}\frac{e^2}{C}$



Inorganica Chimica Acta 361 (2008) 3807–3819

Tunelowanie



letters to nature

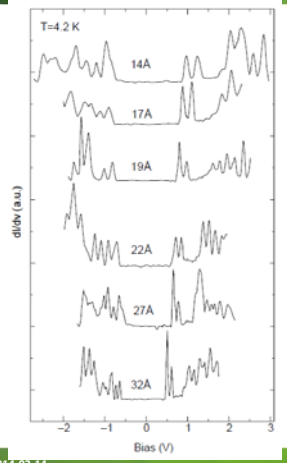
Identification of atomic-like electronic states in indium arsenide nanocrystal quantum dots

Uri Banin*, YunWei Cao*, David Katz† & Oded Millo†

* Department of Physical Chemistry and the Farkas Center for Light Induced Processes, † Raash Institute of Physics, The Hebrew University, Jerusalem 91904, Israel

Figure 1 Scanning tunnelling microscopy and spectroscopy of a single InAs nanocrystal 32 Å in radius, acquired at 4.2 K. The nanocrystal quantum dots (QD) are linked to the gold substrate by hexane dithiol molecules (DT), as shown schematically in the right inset. Left inset, a 10 × 10 nm STM topographic image, showing the nanocrystal. For measuring the *I*–*V* characteristics, the STM tip was positioned above the QD, thus realizing a double-barrier tunnel junction configuration. **a**, The tunnelling *I*–*V* characteristic, exhibiting single-electron tunnelling effects. **b**, The tunnelling conductance spectrum, d*I*/d*V* versus *V*, obtained by numerical differentiation of the *I*–*V* curve (a.u., arbitrary units). The arrows depict the main energy separations: E_c is the single-electron charging energy, E_g is the nanocrystal bandgap, and Δ_{VB} and Δ_{CB} are the spacing between levels in the valence and conduction bands, respectively.

Tunelowanie



letters to nature

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Figure 2 Size evolution of representative tunnelling d*I*/d*V* versus *V* characteristics, displaced vertically. The position of the centre of the observed zero-current gap showed non-systematic variations with respect to zero bias, of the order of 0.2 eV, probably due to variations of local offset potentials. For clarity of presentation, we offset the spectra along the *V* direction to situate the centres of the observed zero-current gaps at zero bias. The nanocrystal radii are denoted in the figure. The range of displayed voltage for each curve reflects the experimental saturation limit of the detected current.

Tunelowanie

„Kolokwium WAN IN"

1. Znajdź w literaturze parametry pasmowe InAs (masy efektywne, powinowactwo elektr., ε_r) [0p]
2. Na podstawie danych z rys. 1 wyznacz pojemność kropki kwantowej o średnicy 32 A. [5p]
3. Na podstawie rys. 2 oraz 3 wyznacz rozmiar nanocząstek InAs w przybliżeniu **nieskończonej** studni potencjału. Porównaj z wartościami mierzonymi oraz z rys. 3. [10p]
4. Na podstawie rys. 2 oraz 3 wyznacz rozmiar nanocząstek InAs w przybliżeniu **skończonej** studni potencjału. Porównaj z wartościami mierzonymi oraz z rys. 3. [30p] (zadanie numeryczne).
5. Oszacuj powinowactwo elektronowe (electron affinity, energię jonizacji z poziomu studni) oraz pojemność kropek kwantowych z rys. 2. (zadanie numeryczne). Porównaj wyniki z danymi tablicowymi (pkt 1) oraz ze wzorem na pojemność kuli i wyznacz ε_r nanocząstki. [30p]

letters to nature

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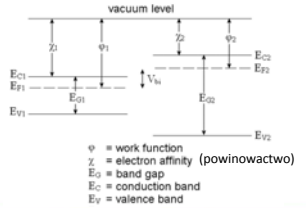

Lokalna gęstość stanów

Gęstość stanów (ogólnie) można zdefiniować jako:

$$N(E) = \sum_n \delta(E - \varepsilon_n)$$

Jak widać po scałkowaniu:

$$\int_{E_1}^{E_2} N(E)\, dE = \int_{E_1}^{E_2} \sum_n \delta(E - \varepsilon_n)\, dE = \sum_n \int_{E_1}^{E_2} \delta(E - \varepsilon_n) dE$$

Przykładowo:

$$N^{1D}(E) = \sum_k \delta\big(E - \varepsilon(k)\big) = \int \frac{1}{E'(k)}\, \delta(k - k')\, 2\, dk = \frac{1}{\pi}\sqrt{\frac{2m}{E}}$$

$$N^{2D}(E) = \sum_k \delta\big(E - \varepsilon(k)\big) = \int \frac{1}{E'(k)}\, \delta(k - k')\, 2\pi k\, dk = \frac{m}{\pi\hbar^2}$$

$$N^{3D}(E) = \sum_k \delta\big(E - \varepsilon(k)\big) = \int \frac{1}{E'(k)}\, \delta(k - k')\, 4\pi k^2\, dk = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{E}$$

Na ćwiczeniach

Jednorodne pole magnetyczne

Rozwiązanie w cechowaniu Landaua:

$$\left\{\frac{1}{2m}\left[\hat{p} - q\,\vec{A}(\vec{r},t)\right]^2 + q\varphi(\vec{r},t) + U(\vec{r},t)\right\}\psi(\vec{r},t) = i\hbar\frac{d}{dt}\psi(\vec{r},t)$$

Cechowania Landaua: pole $\vec{B} = (0,0,B_z) \Rightarrow B_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$ (niestety wyróżnia kierunek)

$\vec{A} = [0, B_z x, 0]$ czyli $A_y = B_z x \stackrel{\text{def}}{=} Bx$ $\quad q = -e$

Przyjmujemy, że w płaszczyźnie xy nie ma innego potencjału

$$\left\{\frac{1}{2m}\left[-\hbar^2\frac{\partial^2}{\partial x^2} + \left(-i\hbar\frac{\partial}{\partial y} + eBx\right)^2 - \hbar^2\frac{\partial^2}{\partial z^2}\right] + U(z)\right\}\psi(\vec{r}) = E\psi(\vec{r})$$

co daje:

$$\left[-\frac{\hbar^2}{2m}\nabla^2 - \frac{ie\hbar}{m}Bx\frac{\partial}{\partial y} + \frac{(eBx)^2}{2m} + U(z)\right]\psi(\vec{r}) = E\psi(\vec{r})$$

Ślad siły Lorentza

Potencjał paraboliczny!

Jednorodne pole magnetyczne

Symetria względem odwrócenia czasu (*time-reversal invariance, T-symmetry*): jeśli rozwiązaniem równania Schrodingera jest funkcja $\Psi(t)$, to rozwiązaniem musi być także $\Psi^*(-t)$ – tylko w przypadku hamiltonianu rzeczywistego. Dla pola magnetycznego musimy także odwrócić kierunek pola magnetycznego: $\Psi(t,\vec{B}) \rightarrow \Psi^*(-t,-\vec{B})$ (musimy odwrócić znak pędu kinetycznego $[\hat{p} - q\,\vec{A}(\vec{r},t)]$.

Urojona wartość – brak symetrii względem odwrócenia czasu

Potencjał w funkcji z jest niezależny od pozostałych zmiennych (faktoryzacja) → rozważamy więc problem 2D + z

co daje: $$\left[-\frac{\hbar^2}{2m}\nabla^2 - \frac{ie\hbar}{m}Bx\frac{\partial}{\partial y} + \frac{(eBx)^2}{2m} + U(z)\right]\psi(\vec{r}) = E\psi(\vec{r})$$

Ślad siły Lorentza

Potencjał paraboliczny!

Potencjał wektorowy nie zależy od y, można założyć postać funkcji: $u(x)\exp(iky)$

Jednorodne pole magnetyczne

$$\left[-\frac{\hbar^2}{2m}\nabla^2 - \frac{ie\hbar}{m}Bx\frac{\partial}{\partial y} + \frac{(eBx)^2}{2m} + U(z)\right]\psi(\vec{r}) = E\psi(\vec{r})$$

Potencjał wektorowy nie zależy od y, można założyć postać funkcji: $\psi(\vec{r}) = w(z)u(x)\exp(ik_y y)$

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}\,m\,\omega_c^2\left(x + \frac{\hbar k_y}{eB}\right)^2\right]u(x) = \varepsilon u(x) \qquad \omega_c = \left|\frac{eB}{m}\right| \qquad R_c = \frac{v}{\omega_c} = \frac{\sqrt{2mE}}{|eB|}$$

Częstość cyklotronowa

Promień cyklotronowy

Wektor falowy k_y. Co ciekawe w ε NIE MA k_y.

Potencjał paraboliczny przesunięty o $x_k = -\hbar k_y/eB$

Jednorodne pole magnetyczne

$$\left[-\frac{\hbar^2}{2m}\nabla^2 - \frac{ie\hbar}{m}Bx\frac{\partial}{\partial y} + \frac{(eBx)^2}{2m} + U(z)\right]\psi(\vec{r}) = E\psi(\vec{r})$$

Potencjał wektorowy nie zależy od y, można założyć postać funkcji: $\psi(\vec{r}) = w(z)u(x)\exp(ik_y y)$

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Długość magnetyczna: $l_B = \sqrt{\frac{\hbar}{m\omega_c}} = \sqrt{\frac{\hbar}{|eB|}}$ nie zależy od masy cząstki, a TYLKO od pola!

Typowa wartość dla $B = 1.0$ T to $l_B = 26$ nm.

Rozwiązania $\varepsilon_{nk} = \left(n - \frac{1}{2}\right)\hbar\omega_c$ (nie zależą od k_y).

$$\phi_{nk}(x,y) \propto H_{n-1}\left(\frac{x - x_k}{l_B}\right)\exp\left[-\frac{(x-x_k)^2}{2l_B^2}\right]\exp(ik_y y)$$

$n = 1, 2, 3 \ldots$ to kolejne **poziomy Landaua**.

Jednorodne pole magnetyczne

Przypadek 2D

Rozwiązania $\varepsilon_{nk} = \left(n - \frac{1}{2}\right)\hbar\omega_c + E_n$ (nie zależy od k_y).

$$\phi_{nk}(x,y) \propto H_{n-1}\left(\frac{x - x_k}{l_B}\right)\exp\left[-\frac{(x-x_k)^2}{2l_B^2}\right]\exp(ik_y y) \qquad n = 1, 2, 3 \ldots$$

Funkcje falowe to funkcje oscylatora (wzdłuż x, rozmiaru rzędu $l_B/\sqrt{2}$) i fali biegnącej (wzdłuż y) – dziwne, prawda? Dlaczego?

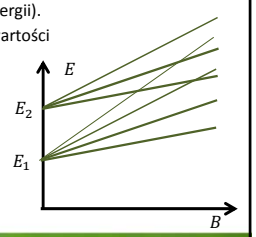
Energia nie zależy od wektora k – stany o różnym k maja tą samą energię, są więc zdegenerowane (a więc dowolna ich kombinacja nie zmienia energii).

Gęstość stanów redukuje się ze stałej $\frac{m}{\pi\hbar^2}$ do serii dyskretnych wartości δ danych przez równanie na ε_{nk} - są to tzw. *poziomy Landaua*.

Pełna energia (łącznie z potencjałem wiążącym w kierunku z):

$$E = E_z + \varepsilon_{nk} = E_z + \left(n - \frac{1}{2}\right)\hbar\omega_c$$

$n = 1, 2, 3 \ldots$



Jednorodne pole magnetyczne

Przypadek 2D

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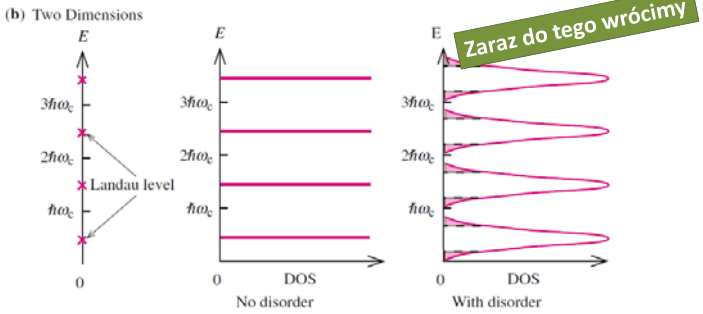


Zaraz do tego wrócimy

Jednorodne pole magnetyczne

Przypadek 2D

Rozwiązania $\varepsilon_{nk} = \left(n - \frac{1}{2}\right)\hbar\omega_c + E_n$ (nie zależy od k_y).

$$\phi_{nk}(x,y) \propto H_{n-1}\left(\frac{x - x_k}{l_B}\right)\exp\left[-\frac{(x-x_k)^2}{2l_B^2}\right]\exp(ik_y y) \qquad n = 1, 2, 3 \ldots$$

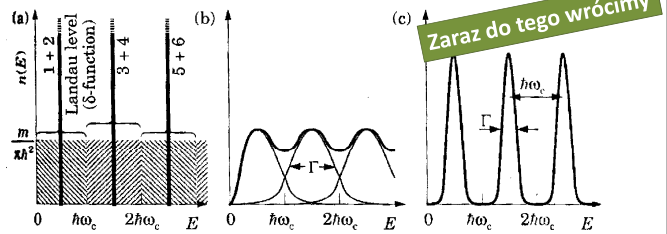


FIGURE 6.7. Density of states in a magnetic field, neglecting spin splitting. (a) The states in each range $\hbar\omega_c$ are squeezed into a δ-function Landau level. (b) Landau levels have a non-zero width Γ in a more realistic picture and overlap if $\hbar\omega_c < \Gamma$. (c) The levels become distinct when $\hbar\omega_c > \Gamma$.

Zaraz do tego wrócimy

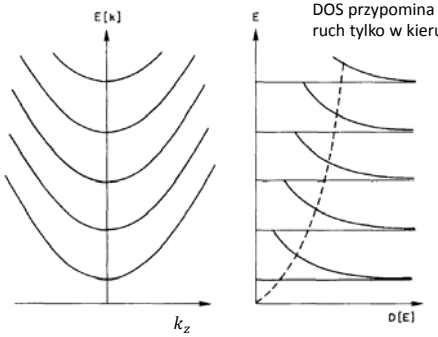
Jednorodne pole magnetyczne

Przypadek 3D (brak potencjału $U(z)$)

Rozwiązania :

$$\varepsilon_{nk} = \left(n - \frac{1}{2}\right)\hbar\omega_c + \frac{\hbar^2 k_z^2}{2m^*} \qquad n = 1, 2, 3 \ldots \text{ to kolejne } \textbf{poziomy Landaua}.$$

DOS przypomina 1D bo jest możliwy ruch tylko w kierunku z



http://www2.warwick.ac.uk/fac/sci/physics/current/postgraduate/regs/mpags/ex5/mag/

Jednorodne pole magnetyczne

Rozwiązanie w cechowaniu symetrycznym:

$$\left\{\frac{1}{2m}\left[\hat{p} - q\,\vec{A}(\vec{r},t)\right]^2 + q\varphi(\vec{r},t) + U(\vec{r},t)\right\}\psi(\vec{r},t) = i\hbar\frac{d}{dt}\psi(\vec{r},t)$$

Cechowania symetryczne: pole $\vec{B} = (0,0,B_z) \Rightarrow A_\theta = \frac{1}{2}Br, A_r = 0, A_z = 0$

$$\left\{-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right] - \frac{i\hbar eB}{m}\frac{\partial}{\partial\theta} + \frac{e^2B^2r^2}{8m} + U(z)\right\}\psi(r,\theta,z) = E\psi(r,\theta,z)$$

Tym razem niezmiennikiem jest obrót o kąt θ, co można powiązać z momentem pędu i funkcją w postaci $\exp(il\theta)$

$$\varepsilon_{nl} = \left(n + \frac{1}{2}l + \frac{1}{2}|l| - \frac{1}{2}\right)\hbar\omega_c \qquad n = 1, 2, 3 \ldots \qquad l = 0, \pm1, \pm2, \pm3 \ldots$$

$$\phi_{nk}(r,\theta) \propto \exp(il\theta)\exp\left[-\frac{r^2}{4l_B^2}\right] r^{|l|} L_{n-1}^{(|l|)}\left(\frac{r^2}{2l_B^2}\right)$$

Potencjał symetryczny też ma swoje wady – gdzie zaczynają się WSZYSTKIE orbity cyklotronowe?

Stowarzyszony wielomian Laguerra

Jednorodne pole magnetyczne

W polu magnetycznym nie możemy zapomnieć o spinie

Spin elektronu: $\mu_B = \frac{e\hbar}{2m_0}$ (magneton Bohra = wielkość momentu magnetycznego elektronu na orbicie o całkowitym momencie pedu $1\hbar$)

$$H' = \mu_B \vec{B} \underline{g} \hat{S}$$

W ogólności g-czynnik może być tensorem

W przypadku elektronu swobodnego $g = 2{,}0023$ …, ale w ciele stałym może mieć bardzo różne wartości (np. $g = -0.44$ w GaAs i $g = +0.4$ w $Al_{0.3}Ga_{0.7}As$).

Jednorodne pole magnetyczne

Wracamy do cechowania Landaua

Rozwiązania $\varepsilon_{nk} = \left(n - \frac{1}{2}\right)\hbar\omega_c + E_n$ (nie zależy od k_y).

$$\phi_{nk}(x,y) \propto H_{n-1}\left(\frac{x - x_k}{l_B}\right)\exp\left[-\frac{(x-x_k)^2}{2l_B^2}\right]\exp(ik_y y) \qquad n = 1, 2, 3 \ldots$$

Pytanie: dla danego n (czyli poziomu Landaua) ile jest różnych stanów $\phi_{nk}(x,y)$ o tej samej energii – czyli jaka jest degeneracja poziomów Landaua?

Policzmy ile jest różnych funkcji o liczbach kwantowych k_y (tylko liczy się k_y, bo w cechowaniu Landaua x_k zależy od k_y) – podobne rozważania można zrobić w dowolnym cechowaniu.

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Jaka jest ilość stanów przypadająca na jeden poziom? Próbka $S = L_x \times L_y$, w cechowaniu Landaua dla współrzędnej y mamy warunek fali płaskiej $k = (2\pi/L_y)n_y$
(n_y to liczba całkowita).

Ile jest wszystkich stanów o różnych n_y?

Dla współrzędnej x funkcja falowa jest centrowana w $x_k = -\frac{\hbar k}{eB} = -(2\pi\hbar n_y/eBL_y)$.
Jeśli n_y będzie zbyt duże, x_k może wyjść poza próbkę .

Poziomy Landaua

Rozwiązanie równania Schrödingera w polu magnetycznym daje widmo dyskretne.

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Dla współrzędnej x funkcja falowa jest centrowana w $x_k = -\frac{\hbar k}{eB} = -(2\pi\hbar n_y/eBL_y)$.

Warunek na to, by x_k było w próbce (a nie poza nią):

$$-L_x < \frac{2\pi\hbar n_y}{eBL_y} < 0 \qquad \text{czyli} \qquad 0 < n_y < \frac{eB}{h} L_x L_y = n_B S \quad \text{(wartość bezwzględna)}$$

Nie ma czynnika 2 związanego z degeneracją spinową (bo w polu magnetycznym spin nie jest zdegenerowany)

Wymiar $n_B = \frac{eB}{h}$ to „ilość" na jednostkę powierzchni

$n_B = \frac{eB}{h}$ degeneracja poziomów Landaua – ilość DOZWOLONYCH stanów na każdym z poziomów Landaua na jednostkę powierzchni – **rośnie z polem** B

Poziomy Landaua

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$$-L_x < \frac{2\pi\hbar n_y}{eBL_y} < 0 \qquad \text{czyli} \qquad 0 < n_y < \frac{eB}{h} L_x L_y = n_B S = \frac{e}{h} BS = \frac{\Phi}{\Phi_0}$$

flux $\Phi_0 = \frac{h}{e} = 4.135667516 \times 10^{-15}$ Wb [Wb]=[T m^2]

„kwant" strumienia magnetycznego (w nadprzewodniku $h/2e$, więc to nie kwant)

$\Phi = BS$ całkowity strumień pola magnetycznego w próbce $S = L_x \times L_y$

$0 < n_y \Phi_0 < \Phi$

Ilość dozwolonych stanów łączy się wiec z ilością „fluksów" przechodzących przez próbkę!

Lokalna gęstość stanów

Gęstość stanów (ogólnie) można zdefiniować jako:

$$N(E) = \sum_n \delta(E - \varepsilon_n)$$

Jak widać po scałkowaniu:

$$\int_{E_1}^{E_2} N(E)\, dE = \int_{E_1}^{E_2} \sum_n \delta(E - \varepsilon_n)\, dE = \sum_n \int_{E_1}^{E_2} \delta(E - \varepsilon_n) dE$$

Przykładowo:

$$N^{1D}(E) = \sum_k \delta\big(E - \varepsilon(k)\big) = \int \frac{1}{E'(k)}\, \delta(k - k')\, 2\, dk = \frac{1}{\pi}\sqrt{\frac{2m}{E}}$$

$$N^{2D}(E) = \sum_k \delta\big(E - \varepsilon(k)\big) = \int \frac{1}{E'(k)}\, \delta(k - k')\, 2\pi k\, dk = \frac{m}{\pi\hbar^2}$$

$$N^{3D}(E) = \sum_k \delta\big(E - \varepsilon(k)\big) = \int \frac{1}{E'(k)}\, \delta(k - k')\, 4\pi k^2\, dk = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{E}$$

Na ćwiczeniach

Poziomy Landaua

$N^{2D}(E) = \frac{m}{\pi\hbar^2}$ Poszerzenie poziomów na skutek rozproszeń $\Gamma = \hbar/\tau_i$

τ_i to jednocząstkowy (albo kwantowy) czas życia – to NIE jest ten sam czas, który omawialiśmy w modelu Drudego (transportowy)

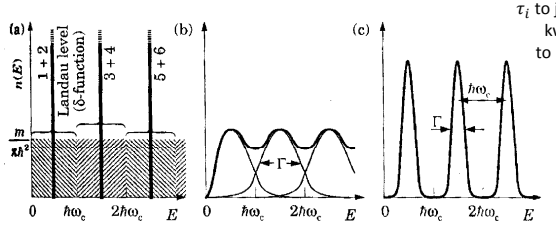

FIGURE 6.7. Density of states in a magnetic field, neglecting spin splitting. (a) The states in each range $\hbar\omega_c$ are squeezed into a δ-function Landau level. (b) Landau levels have a non-zero width Γ in a more realistic picture and overlap if $\hbar\omega_c < \Gamma$. (c) The levels become distinct when $\hbar\omega_c > \Gamma$.

Licząc 2 spiny: $2n_B = \frac{2eB}{h} = \frac{2m\omega_c}{2\pi\hbar} = \frac{m}{\pi\hbar^2}\,\hbar\omega_c$

Każdy ze stanów na poziome Landaua zajmuje powierzchnię $\frac{h}{eB} = 2\pi l_B^2$

$l_B = \sqrt{\frac{\hbar}{m\omega_c}} = \sqrt{\frac{\hbar}{|eB|}}$

Poziomy Landaua

$n_B = \frac{eB}{h}$ degeneracja poziomów Landaua – ilość DOZWOLONYCH stanów na każdym z poziomów Landaua na jednostkę powierzchni – **rośnie z polem** B

Koncentracja nośników 2D: n_{2D} – na ilu poziomach Landaua zmieszczą się te nośniki?

Współczynnik wypełnienia *filling factor* ν (zwykle nie jest to liczba całkowita)

$$\nu = \frac{n_{2D}}{n_B} = \frac{hn_{2D}}{eB} = \frac{\Phi_0 n_{2D}}{B} = 2\pi l_B^2 n_{2D}$$ (z uwzględnieniem degeneracji spinów)

Zwiększając pole magnetyczne kolejno zapełniamy poziomy Landaua. Można całkowicie zapełnić n-ty poziom ($\nu = n$) i wtedy $B_n = hn_{2D}/en$, aż osiągniemy $n = 1$, czyli wszystkie elektrony będą na tym samym poziomie Landaua (tzw. *limit kwantowy*).

Dla $\nu < 1$ zaczynają się dziać ciekawe rzeczy (do których zaraz wrócimy!)

Poziomy Landaua

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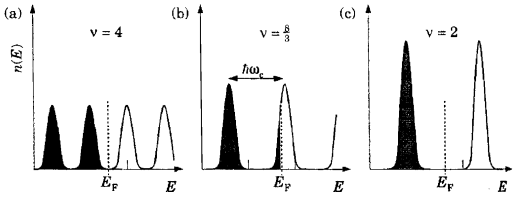
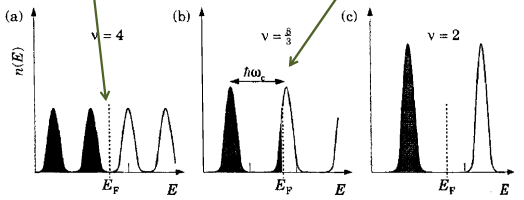


FIGURE 6.8. Occupation of Landau levels in a magnetic field neglecting the spin splitting, showing how the Fermi level moves to maintain a constant density of electrons. The fields are in the ratio 2 : 3 : 4 and give $\nu = 4, \frac{8}{3}$, and 2.

Poziomy Landaua

Poziom Fermiego leży **pomiędzy** poziomami Landaua – nie ma tam DOS, zmiana E_F nie zmienia DOS – stany *nieściśliwe* (*incompressible*)

Poziom Fermiego leży **wewnątrz** poziomu Landaua – duża DOS, zmiana E_F mocno wpływa na DOS – stany *ściśliwe* (*compressible*)



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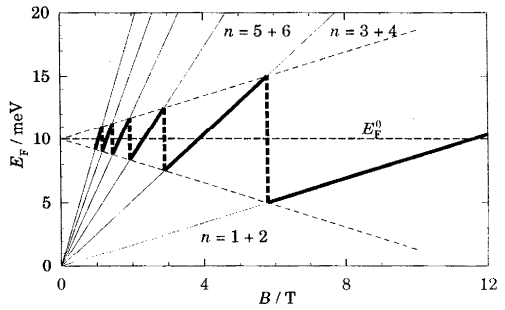

FIGURE 6.9. Variation of the Fermi level as a function of magnetic field for a two-dimensional electron gas in GaAs with $E_F^0 = 10$ meV before the field was applied. Spin splitting is neglected. The fan of thin lines shows the Landau levels, while the discontinuous thick line is E_F.

Poziomy Landaua

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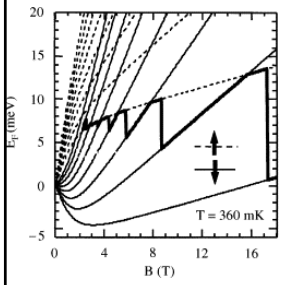

Fig. 16. Landau level fan diagram for the magnetic 2DEG sample described in Fig. 15. Solid (dashed) lines correspond to spin-down (spin-up) states. The dark solid line shows the variation of the Fermi energy with magnetic field. Parameters used in this calculation are: E_F=7 meV at B=0, and T=360 mK. The spin-splitting parameters used are obtained by fitting the magneto-optical data in Fig. 3: T_0=2.1 K and a saturation conduction band spin splitting of 12.9 meV.

Spin dynamics and quantum transport in magnetic semiconductor quantum structures
D.D Awschalom, N. Samarth, Journal of Magnetism and Magnetic Materials **200** (1999) 130-147

Efekt Shubnikova-de Haasa

Shubnikov-de Haas effect

9.4.1 Types of quantum oscillation

As the electronic density of states at E_F determines most of a metal's properties, virtually all properties will exhibit quantum oscillations in a magnetic field. Examples include[7]

- oscillations of the magnetisation (the de Haas–van Alphen effect);
- oscillations of the magnetoresistance (the Shubnikov–de Haas effect);
- oscillations of the sample length;
- ocillations of the sample temperature;
- oscillations in the ultrasonic attenuation;
- oscillations in the Peltier effect and thermoelectric voltage;
- oscillations in the thermal conductivity.

[6]However, open orbits do lead to a very interesting quantum phenomenon which has recently been observed in high-frequency experiments; see A. Ardavan *et al., Phys. Rev. B* **60**, 15500 (1999); *Phys. Rev. Lett.* **81**, 713 (1998).
[7]Some pictures of typical data are shown in *Solid State Physics*, by N.W Ashcroft and N.D. Mermin (Holt, Rinehart and Winston, New York 1976) pages 266-268.

http://www2.physics.ox.ac.uk/sites/default/files/BandMT_09.pdf

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Gęstość stanów oscyluje - spada do 0 dla $\nu = n$ i jest największa dla $\nu \approx n + \frac{1}{2}$ - najprościej to zmierzyć w magnetooporze R_{xx}.

Oscylacje zależą od stodunku energii Fermiego E_F do częstości cyklotronowej $\hbar\omega_c = eB/m^*$. Oscylacje są periodyczne w funkcji $1/B$.

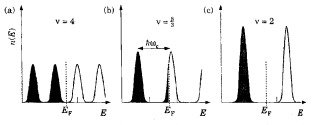
$$\nu = \frac{n_{2D}}{n_B} = \frac{hn_{2D}}{eB} = \frac{\Phi_0 n_{2D}}{B} = 2\pi l_B^2 n_{2D}$$

Z SdH można także wyznaczyć masę efektywną m^* oraz czas kwantowy τ_q. Amplituda oscylacji jest dana przez

$$\Delta\rho_{SdH} = 4\rho_0\delta\cos(4\pi\nu)\frac{\xi(T)}{\sinh(\xi(T))}\exp\left(-\frac{\pi}{\omega_c\tau_q}\right)$$

$$\xi(T) = 2\pi^2 kT/\hbar\omega_c$$

Zależność temperaturowa daje m^*, tłumienie τ_q.



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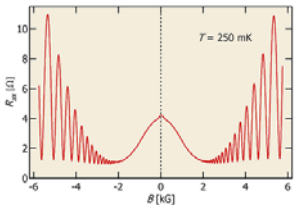

http://groups.physics.umn.edu/zudovlab/content/sdho.htm

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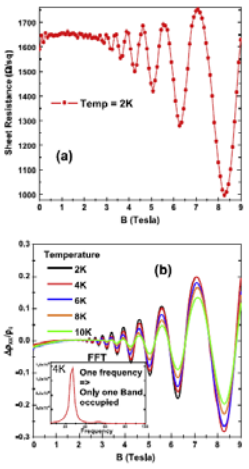


A. Nainani et al. Solid-State Electronics 62 (2011) 138–141

Efekt Shubnikova-de Haa

Shubnikov-de Haas effect

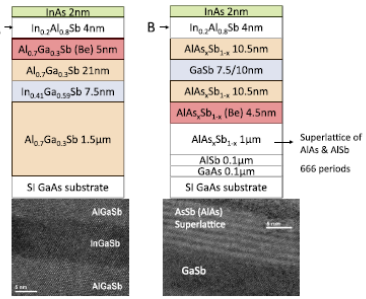
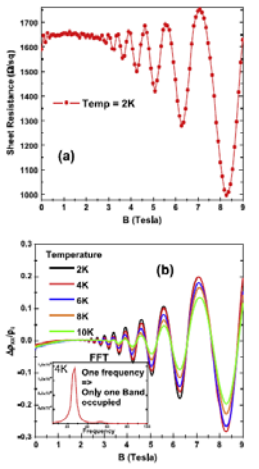


Fig. 1. Cross-section showing the layer details in a quantum well heterostructure with (A) $In_xGa_{1-x}Sb$ and (B) GaSb channel. The $AlAs_ySb_{1-y}$ layers are composed of AlSb/AlAs short-period superlattices. Also shown are high resolution TEM images around the channel region.



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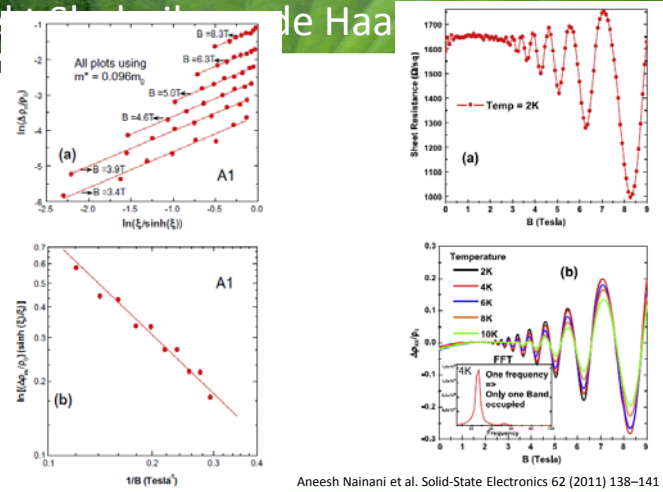

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Efekt Shubnikova-de Haa

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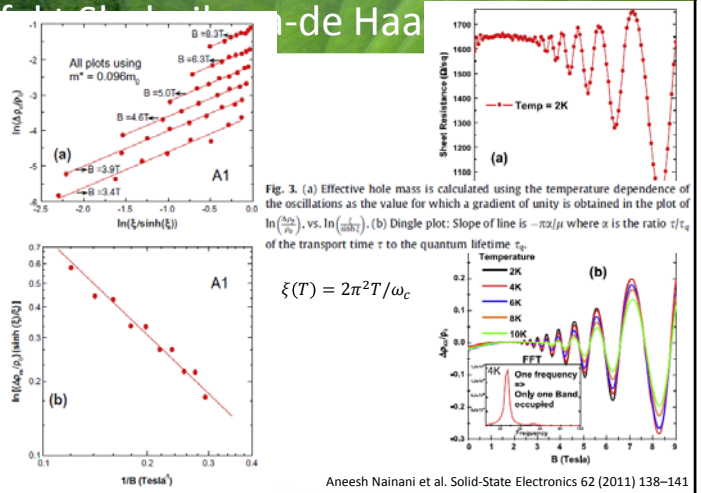

Fig. 3. (a) Effective hole mass is calculated using the temperature dependence of the oscillations as the value for which a gradient of unity is obtained in the plot of $\ln\left(\frac{\Delta\rho_{xx}}{\rho_0}\right)$ vs. $\ln\left(\frac{\xi}{\sinh\xi}\right)$. (b) Dingle plot: Slope of line is $-\pi\alpha/\mu$ where α is the ratio τ/τ_q of the transport time τ to the quantum lifetime τ_q.

$$\xi(T) = 2\pi^2 T/\omega_c$$

Aneesh Nainani et al. Solid-State Electronics 62 (2011) 138–141

Efekt Shubnikova-de Haasa

Shubnikov-de Haas effect

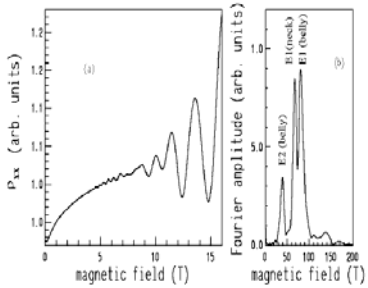

Figure 1. Shubnikov-de Haas oscillations (a) and its Fourier transform (b) measured at 4.2K for sample No.326.

Henriques et al. Brazil. J. of Phys. 29, 707 (1999)

Efekt Shubnikova-de Haasa

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Gęstość stanów oscyluje - spada do 0 dla $\nu = n$ i jest największa dla $\nu \approx n + \frac{1}{2}$ - najprościej to zmierzyć w magnetooporze R_{xx}.

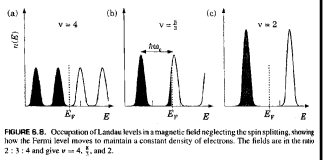
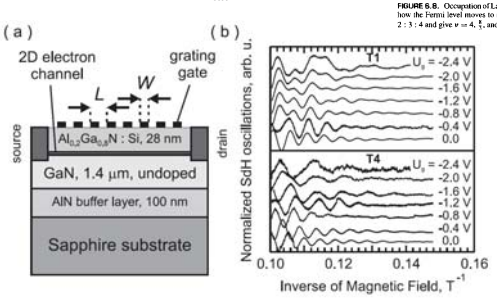


FIGURE 6.8. Occupation of Landau levels in a magnetic field neglecting the spin splitting, showing how the Fermi level moves to maintain a constant density of electrons. The fields are in the ratio 2 : 3 : 4 and give $\nu = 4, \frac{8}{3}$, and 2.

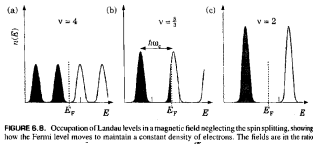


K. Nogajewski et al., Appl. Phys. Lett. 99, 213501 (2011)

Efekt Shubnikova-de Haasa

Shubnikov-de Haas effect

Gęstość stanów oscyluje - spada do 0 dla $\nu = n$ i jest największa dla $\nu \approx n + \frac{1}{2}$ - najprościej to zmierzyć w magnetooporze R_{xx}.



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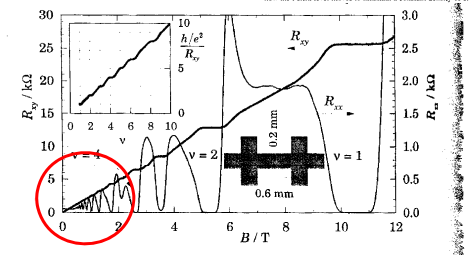
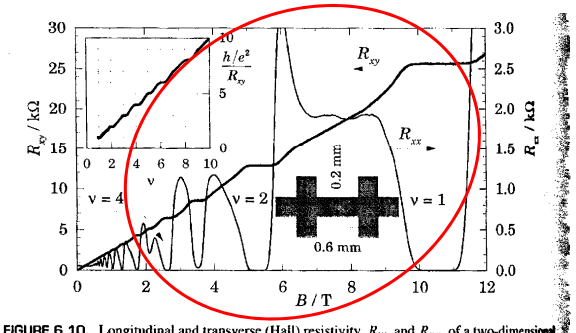


FIGURE 6.10. Longitudinal and transverse (Hall) resistivity, R_{xx} and R_{xy}, of a two-dimensional electron gas of density $n_{2D} = 2.6 \times 10^{15}\,\mathrm{nm}^{-2}$ as a function of magnetic field. The measurements were made at $T = 1.13\,\mathrm{K}$. The inset shows $1/R_{xx}$ divided by the quantum unit of conductance e^2/h as a function of the filling factor ν. [Data kindly supplied by Dr A. R. Long, University of Glasgow.]

Kwantowy (całkowity) efekt Halla (IQHE)

Integer Quantum Hall effect (IQHE) – dla gazu 2D jeśli poziom Fermiego znajduje się w obszarze stanów zlokalizowanych opór hallowski jest skwantowany

$$R_H = \frac{1}{\nu}\frac{h}{e^2}$$



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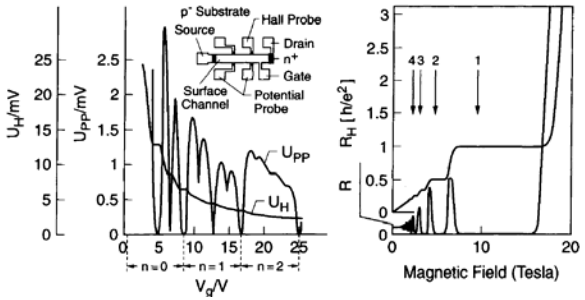

Figure 7. Left panel: original data of the discovery of the integral quantum Hall effect (IQHE) by Klaus von Klitzing in 1980 in the two-dimensional electron system of a silicon MOSFET transistor. Instead of a smooth curve he observed plateaus in the Hall voltage (U_H) and found concomitant deep minima in the magneto resistance (U_{PP}). The horizontal axis represents gate voltage (V_G) which varies the carrier density, n. The right panel shows equivalent data taken on a two-dimensional electron system in GaAs/AlGaAs. Since these data are plotted versus magnetic field they can directly be compared to Edwin Hall's data of Fig. 6. Rather than the linear dependence of the Hall resistance on magnetic field of Fig. 6, these data show wide plateaus in R_H and in addition deep minima in R.

Stromer, Nobel Lecture

Tensor przewodnictwa

Przewodnictwo $\sigma = n\,e\,\mu$

Gęstość prądu: $\vec{j} = \sigma\vec{E}$ - w ogólności σ może być tensorem:

$\vec{j}_x = q\,n\,\vec{v}_x$ oraz $\vec{v}_x = \frac{q\tau}{m}\vec{E}_x = \mu\,\vec{E}_x$

W ogólności np. $j_x = \sigma_{xx}\,E_x$ oraz $j_y = \sigma_{yx}E_x$ itp.

Model Drudego z polem magnetycznym: $m^*\left\{\frac{d\vec{v}}{dt} + \frac{\vec{v}}{\tau}\right\} = q\vec{E} + q\vec{v}\times\vec{B}$

τ – czas relaksacji pędowej (*scattering time*)

$$m^*\left\{\frac{dv_x}{dt} + \frac{v_x}{\tau}\right\} = qE_x + qv_yB$$

$$m^*\left\{\frac{dv_y}{dt} + \frac{v_y}{\tau}\right\} = qE_y - qv_xB$$

http://www2.physics.ox.ac.uk/sites/default/files/BandMT_11.pdf

Efekt Halla

Siła Lorentza: $\vec{F} = q\vec{v} \times \vec{B}$

Model Drudego: $m^*\left\{\frac{d\vec{v}}{dt} + \frac{\vec{v}}{\tau}\right\} = q\vec{E} + q\vec{v} \times \vec{B}$

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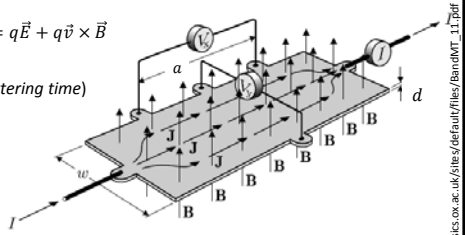


Dostajemy:

$$v_y\{1 + \omega_c^2\tau^2\} = \frac{q\tau}{m^*}\left(E_y - \omega_c\tau E_x\right)$$

$$\omega_c = \frac{qB}{m^*} \qquad \mu = \frac{q\tau}{m^*} \qquad j_y = 0 = \sum_i q_i n v_y^i$$

Na ćwiczeniach

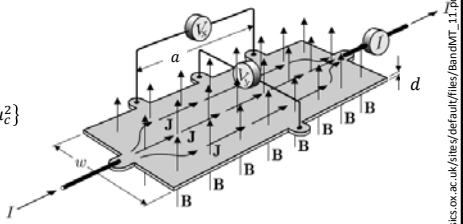
http://www2.physics.ox.ac.uk/sites/default/files/BandMT_11.pdf

Efekt Halla

Zaniedbując $\omega_c^2\tau^2 \ll 1$ i biorąc pod uwagę przewodnictwo elektronów n i dziur p:

$$j_y = 0 = \sum_i q_i n v_y^i$$

$$E_y\{n\mu_c + p\mu_h\} = E_xB\{p\mu_h^2 - n\mu_c^2\}$$



Dostajemy tzw. stałą Halla:

$$R_H = \frac{E_y}{j_xB} = \frac{1}{|e|}\frac{p\mu_h^2 - n\mu_c^2}{(n\mu_c + p\mu_h)^2}$$

Np. dla $p = 0$ mamy $R_H = -\frac{1}{en}$

Na ćwiczeniach

http://www2.physics.ox.ac.uk/sites/default/files/BandMT_11.pdf

Efekt Halla

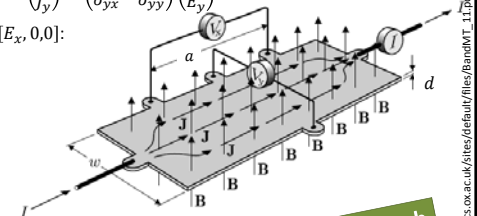
Biorąc $\sigma_0 = n\,e\,\mu = n\,e^2\,\tau/m$ $\quad \begin{pmatrix} J_x \\ J_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}\begin{pmatrix} E_x \\ E_y \end{pmatrix}$

W przypadku efektu Halla $E = [E_x, 0, 0]$:

$$j_x = \sigma_{xx}E_x \qquad j_y = \sigma_{yx}E_x$$

$$\sigma_{xx} = \frac{\sigma_0}{1 + \omega_c^2\tau^2}$$

$$\sigma_{yx} = \frac{\sigma_0\omega_c\tau}{1 + \omega_c^2\tau^2}$$



Na ćwiczeniach

Tensor przewodnictwa:

$$\sigma = \begin{pmatrix} \sigma_{xx} & -\sigma_{yx} \\ \sigma_{yx} & \sigma_{xx} \end{pmatrix} = \begin{pmatrix} \sigma_L & -\sigma_T \\ \sigma_T & \sigma_L \end{pmatrix} = \frac{\sigma_0}{1 + \omega_c^2\tau^2}\begin{pmatrix} 1 & \omega_c\tau \\ -\omega_c\tau & 1 \end{pmatrix}$$

Tensor oporności: $\rho = \frac{1}{\sigma_L^2 + \sigma_T^2}\begin{pmatrix} \sigma_L & \sigma_T \\ -\sigma_T & \sigma_L \end{pmatrix} = \frac{1}{\sigma_0}\begin{pmatrix} 1 & \omega_c\tau \\ -\omega_c\tau & 1 \end{pmatrix}$

http://www2.physics.ox.ac.uk/sites/default/files/BandMT_11.pdf

Efekt Halla

Pełny tensor przewodnictwa

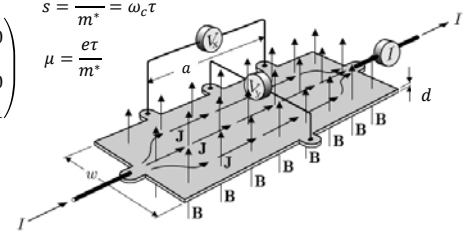
$$\sigma = ne\mu\begin{pmatrix} \frac{1}{1+s^2} & \frac{-s}{1+s^2} & 0 \\ \frac{s}{1+s^2} & \frac{1}{1+s^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$s = \frac{eB\tau}{m^*} = \omega_c\tau$$

$$\mu = \frac{e\tau}{m^*}$$

Pełny tensor oporności

$$\rho = \sigma^{-1} = \frac{1}{ne\mu}\begin{pmatrix} 1 & s & 0 \\ -s & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



$$\vec{E} = \rho\vec{j} = \begin{bmatrix} \frac{j_x}{ne\mu} \\ -\frac{j_xB}{ne} \\ 0 \end{bmatrix}$$

$$U_{xy} = E_yw = \frac{I_x}{wd}\frac{B}{ne}w = \frac{I_x}{dne}B = R_H\frac{I_xB}{d}$$

$$R_H = \frac{1}{ne}$$ stała Halla

Efekt Halla

Pełny tensor przewodnictwa

$$\sigma = ne\mu \begin{pmatrix} \frac{1}{1+s^2} & \frac{-s}{1+s^2} & 0 \\ \frac{s}{1+s^2} & \frac{1}{1+s^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Pełny tensor oporności

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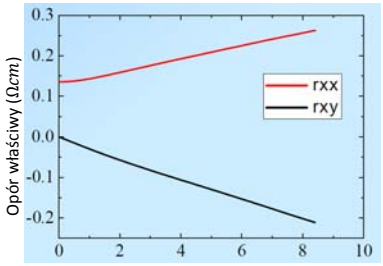


Dla różnych kanałów przewodnictwa

$\sigma = \sum_i \sigma_i$ transport wielonośnikowy – analizujemy tensor σ

Roman Stępniewski

Efekt Halla

Pełny tensor przewodnictwa

$$\sigma = ne\mu \begin{pmatrix} \frac{1}{1+s^2} & \frac{-s}{1+s^2} & 0 \\ \frac{s}{1+s^2} & \frac{1}{1+s^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

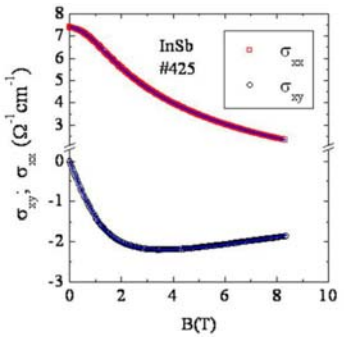
Pełny tensor oporności

$$\rho = \sigma^{-1} = \frac{1}{ne\mu} \begin{pmatrix} 1 & s & 0 \\ -s & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Dla różnych kanałów przewodnictwa

$\sigma = \sum_i \sigma_i$ transport wielonośnikowy

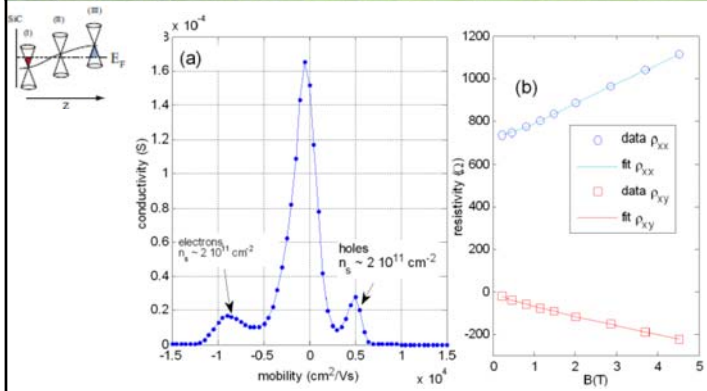
$R_H = \frac{1}{ne}$



$n_1 = (1.483 \pm .004)10^{16}$ $\mu_1 = (1361 \pm 5)$ cm²/Vs
$n_2 = (4.60 \pm .02)10^{15}$ $\mu_2 = (4622.6 \pm 9.5)$ cm²/Vs
$p = (1.77 \pm .06)\ 10^{16}$ $\mu_h = (255 \pm 7)$ cm²/Vs

Roman Stępniewski

Efekt Halla



Transport wielonośnikowy w grafenie (M. Gryglas-Borysiewicz)

Roman Stępniewski

Efekt Halla

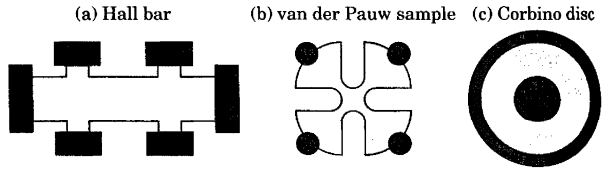


FIGURE 6.5. Samples commonly used for measuring the conductivity of semiconductors: (a) Hall bar, (b) van der Pauw sample, and (c) Corbino disc. The dark areas are the contacts for measuring voltage or current, and the light areas are the active regions of the sample.

$$\sigma = ne\mu \begin{pmatrix} \frac{1}{1+s^2} & \frac{-s}{1+s^2} & 0 \\ \frac{s}{1+s^2} & \frac{1}{1+s^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$s = \frac{eB\tau}{m^*} = \omega_c \tau$

$\mu = \frac{e\tau}{m^*}$

Efekt Halla - kwantowy

The Nobel Prize in Physics 1985
Klaus von Klitzing

The Nobel Prize in Physics 1985
Nobel Prize Award Ceremony
Klaus von Klitzing

Klaus von Klitzing

The Nobel Prize in Physics 1985 was awarded to Klaus von Klitzing *"for the discovery of the quantized Hall effect"*.

Efekt Halla - kwantowy

The Nobel Prize in Physics 1998
Robert B. Laughlin, Horst L. Störmer, Daniel C. Tsui

The Nobel Prize in Physics 1998
Nobel Prize Award Ceremony
Robert B. Laughlin
Horst L. Störmer
Daniel C. Tsui

Robert B. Laughlin Horst L. Störmer Daniel C. Tsui

The Nobel Prize in Physics 1998 was awarded jointly to Robert B. Laughlin, Horst L. Störmer and Daniel C. Tsui *"for their discovery of a new form of quantum fluid with fractionally charged excitations"*.

Efekt Halla - kwantowy

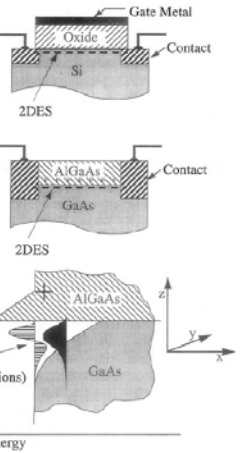
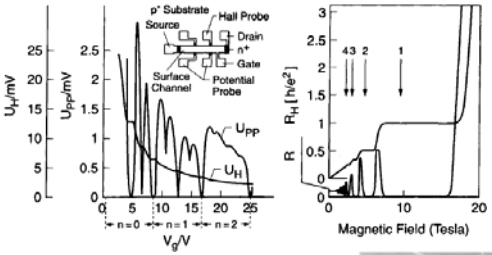
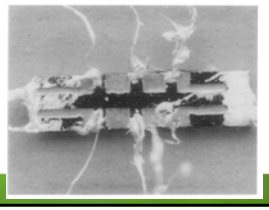


Figure 1 a). Schematic drawings of a silicon Metal Oxide Semiconductor Field Effect Transistor (MOSFET). The two-dimensional electron system (2DES) resides at the interface between silicon and silicon oxide. Electrons are held against the oxide by the electric field from the gate metal. b) Schematic drawings of a modulation-doped gallium arsenide/aluminum gallium arsenide (GaAs/AlGaAs) heterojunction. The 2DES resides at the interface between GaAs and AlGaAs. Electrons are held against the AlGaAs by the electric field from the charged silicon dopants (+) in the AlGaAs. c). Energetic condition in the modulation-doped structure (very similar to the condition in the MOSFET). Energy increases to the left. Electrons are trapped in the triangular-shaped quantum-well at the interface. They assume discrete energy states in the z-direction (black and horizontally striped). At low temperatures and low electron concentration only the lowest (black) electron state is occupied. The electrons are totally confined in the z-direction but can move freely in the x-y-plane.

Horst Stormer, *Nobel Lecture*

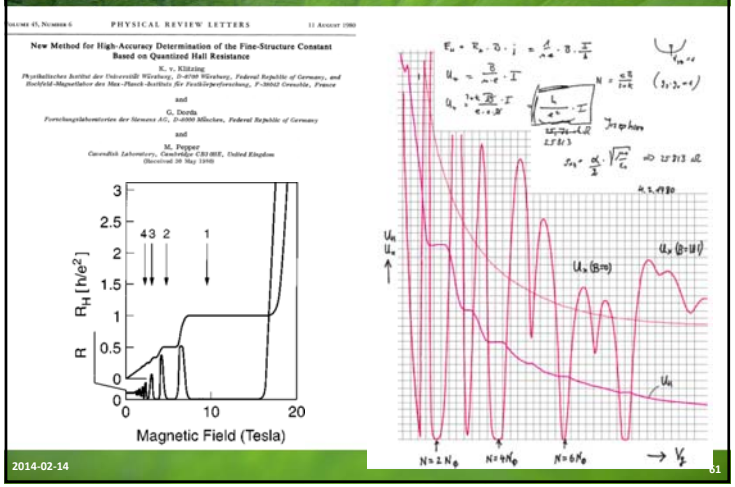
Efekt Halla - kwantowy





Horst Stormer, *Nobel Lecture*

Efekt Halla - kwantowy



Efekt Halla - kwantowy

Pełny tensor przewodnictwa

$$\sigma = \begin{pmatrix} \sigma_L & -\sigma_T \\ \sigma_T & \sigma_L \end{pmatrix} = \frac{\sigma_0}{1+\omega_c^2\tau^2}\begin{pmatrix} 1 & \omega_c\tau \\ -\omega_c\tau & 1 \end{pmatrix}$$

Pełny tensor oporności

$$\rho = \sigma^{-1} = \frac{1}{ne\mu}\begin{pmatrix} 1 & -s & 0 \\ s & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\rho = \frac{1}{\sigma_L^2+\sigma_T^2}\begin{pmatrix} \sigma_L & \sigma_T \\ -\sigma_T & \sigma_L \end{pmatrix} = \frac{1}{\sigma_0}\begin{pmatrix} 1 & \omega_c\tau \\ -\omega_c\tau & 1 \end{pmatrix}$$

Dla dużych pól magnetycznych $|\sigma_T| \gg \sigma_L$

$$\rho = \frac{1}{\sigma_L^2+\sigma_T^2}\begin{pmatrix} \sigma_L & \sigma_T \\ -\sigma_T & \sigma_L \end{pmatrix} \approx \begin{pmatrix} \sigma_L/\sigma_T^2 & -1/\sigma_T \\ 1/\sigma_T & \sigma_L/\sigma_T^2 \end{pmatrix}$$

wtedy $\rho \sim \sigma_L$!

Efekt Halla

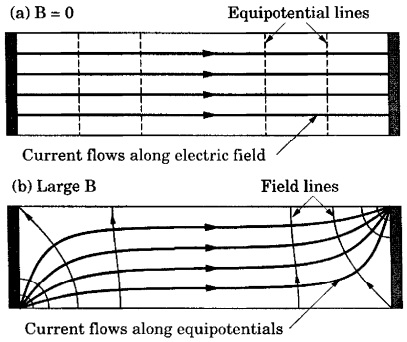


FIGURE 6.6. Electric field, current flow, and equipotentials inside a long rectangular sample with contacts across each end. (a) In the absence of an electric field the current is uniform throughout the sample and runs along the electric field. (b) In a strong magnetic field, where $|\sigma_T| \gg \sigma_L$, the current runs along equipotentials.

Kwant przewodnictwa

$$G = \frac{dI}{dU} = \frac{2e^2}{h}\int_{E_L}^{\infty} \frac{\partial f(E,\mu)}{\partial E} T(E)dE \approx \frac{2e^2}{h}T(\mu) = G_0 T(\mu)$$

$$\frac{e^2}{h} = 38{,}7\ \mu S$$

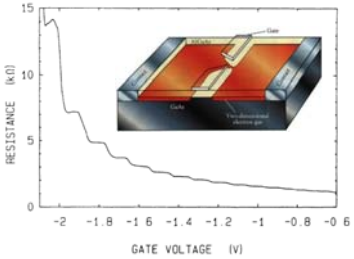


FIG. 1. Point-contact resistance as a function of gate voltage at 0.6 K. Inset: Point-contact layout.

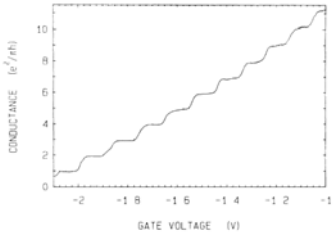


FIG. 2. Point-contact conductance as a function of gate voltage, obtained from the data of Fig. 1 after the lead resistance. The conductance is plotted in units of $e^2/\pi h$.

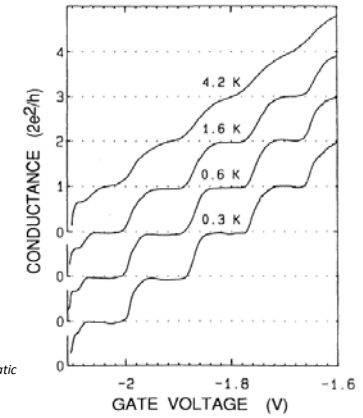
B. J. van Wees et al. *Quantized conductance of point contacts in a two-dimensi*
Phys. Rev. Lett. **60**, 848–850 (1988)

Przypomnienie

Kwant przewodnictwa

$$G = \frac{2e^2}{h} T(\mu) = G_0 T(\mu)$$

Przypomnienie



B. J. van Wees et al. *Quantum ballistic and adiabatic electron transport studied with quantum point contacts* Phys. Rev. B 43, 12431–12453 (1991)

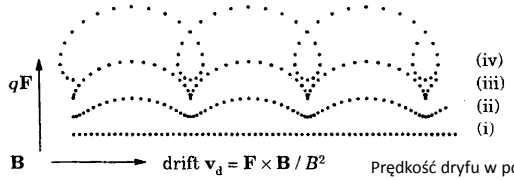
FIG. 6. Breakdown of the conductance quantization due to temperature averaging. The curves have been offset for clarity.

Pole elektrycznie i magnetyczne

Ruch elektronu w skrzyżowanych polach elektrycznym $\vec{E} = (E, 0, 0)$ i magnetycznym $\vec{B} = (0, 0, B)$ opisany jest cykloidą:

$$x(t) = -\frac{mE}{eB^2}(1 - \cos\omega_c t)$$
$$y(t) = -\frac{mE}{eB^2}(\omega_c t - \sin\omega_c t)$$

Szczegóły ruchu zależą od warunków początkowych



Prędkość dryfu w polu B: $v_d = \frac{E}{B}$

FIGURE 6.11. Classical motion of a charged particle in crossed electric and magnetic fields, with **B** normal to the page and equal intervals of time between the symbols. The curves correspond to different initial velocities and energies, with (iii) showing the cycloid for a particle initially at rest.

Jednorodne pole magnetyczne

$$\left[-\frac{\hbar^2}{2m}\nabla^2 - \frac{ie\hbar}{m}Bx\frac{\partial}{\partial y} + \frac{(eBx)^2}{2m} + U(z)\right]\psi(\vec{r}) = E\psi(\vec{r})$$

Potencjał wektorowy nie zależy od y, można założyć postać funkcji: $\psi(\vec{r}) = w(z)u(x)\exp(ik_y y)$

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega_c^2\left(x + \frac{\hbar k_y}{eB}\right)^2\right]u(x) = \varepsilon u(x) \qquad \omega_c = \left|\frac{eB}{m}\right| \qquad R_c = \frac{v}{\omega_c} = \frac{\sqrt{2mE}}{|eB|}$$

Częstość cyklotronowa

Promień cyklotronowy

Wektor falowy k_y. Co ciekawe w ε NIE MA k_y.

Potencjał paraboliczny przesunięty o $x_k = -\hbar k_y/eB$

Przypomnienie

Pole elektrycznie i magnetyczne

$$\left[-\frac{\hbar^2}{2m}\nabla^2 - \frac{ie\hbar}{m}Bx\frac{\partial}{\partial y} + \frac{(eBx)^2}{2m} + eEx\right]\psi(\vec{r}) = E\psi(\vec{r})$$

Potencjał wektorowy nie zależy od y, można założyć postać funkcji: $\psi(\vec{r}) = w(z)u(x)\exp(ik_y y)$

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega_c^2\left(x + \frac{\hbar k_y}{eB} + \frac{Ee}{m\omega_c^2}\right)^2 - \frac{\hbar kE}{B} - \frac{mE^2}{2B^2}\right]u(x) = \varepsilon u(x)$$

Człony „dołożone" żeby suma w kwadracie dawała eE

Potencjał paraboliczny przesunięty o $x_k = -\left(\frac{\hbar k_y}{eB} + \frac{Ee}{m\omega_c^2}\right) = \frac{mv_d - \hbar k}{eB}$

$$\varepsilon_{nk} = \left(n - \frac{1}{2}\right)\hbar\omega_c - \frac{\hbar kE}{B} - \frac{mE^2}{2B^2} = \left(n - \frac{1}{2}\right)\hbar\omega_c - eEx_k - \frac{1}{2}mv_D^2$$

$$v_d = \frac{E_x}{B_z}$$

$J_y = -en_{2D}v_D = en_{2D}\frac{E_x}{B_z} \Rightarrow \sigma_{xx} = \sigma_L = 0$ $\qquad \rho_T = 1/\sigma_T = B_z/en_{2D}$ (klasyczny efekt Halla)

Pole elektrycznie i magnetyczne

Potencjał paraboliczny zależy od pola i od k_y

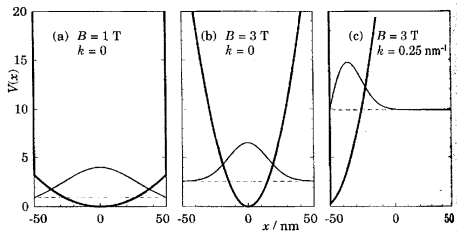


FIGURE 6.13. Potential energy and lowest eigenstate in a magnetic field for an electron with wave number k in a hard-walled wire of width 0.1 μm in GaAs.

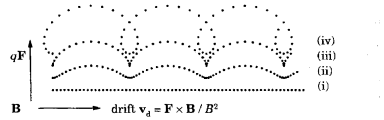


FIGURE 6.11. Classical motion of a charged particle in crossed electric and magnetic fields, with B normal to the page and equal intervals of time between the symbols. The curves correspond to different initial velocities and energies, with (iii) showing the cycloid for a particle initially at rest.

$$v_d = \frac{E}{B}$$

Pole elektrycznie i magnetyczne

Potencjał paraboliczny zależy od pola i od k_y

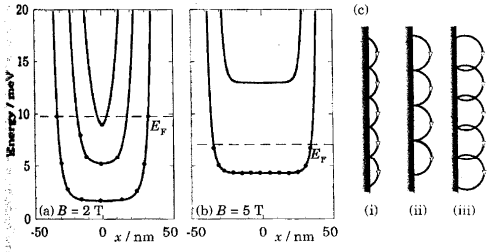


FIGURE 6.14. Energies $\varepsilon_{nk}(B)$ of electrons in a hard-walled wire in a magnetic field of (a) 2 T and (b) 5 T, plotted against the guiding centre (x_{nk}). For clarity, only the lowest three bands are shown for $B = 2$ T. The dots represent some of the occupied states and the dashed line is the Fermi level. (c) Classical skipping orbits along the edge of a wire.

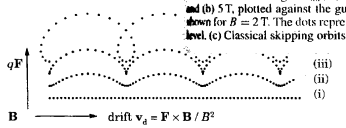


FIGURE 6.11. Classical motion of a charged particle in crossed electric and magnetic fields, with B normal to the page and equal intervals of time between the symbols. The curves correspond to different initial velocities and energies, with (iii) showing the cycloid for a particle initially at rest.

$$v_d = \frac{E}{B}$$

Pole elektrycznie i magnetyczne

Potencjał paraboliczny zależy od pola i od k_y

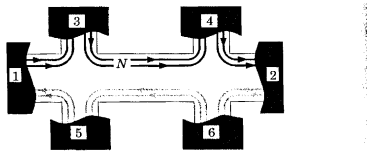


FIGURE 6.18. A Hall bar in a strong magnetic field, showing the propagation of edge states. A negative bias on contact 1 injects extra electrons into the N edge states that leave it (only two of which are drawn); the electrons depart through the other current probe (2).

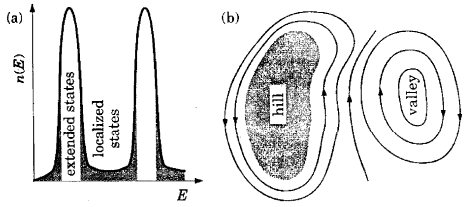


FIGURE 6.19. (a) Density of states of a Landau level in a disordered system showing a band of extended states in the centre of each level with localized states in between. (b) Edge states localized in a slowly varying potential, with a hill on the left and a hollow on the right.

Pole elektrycznie i magnetyczne

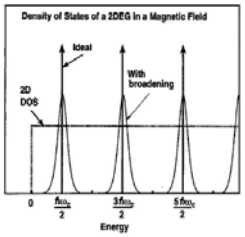


Figure 5.2: Density of states of a 2DEG in magnetic field. By comparing the DOS with and without magnetic field, we can calculate the number of states within each Landau level. From Ferry & Goodnick.

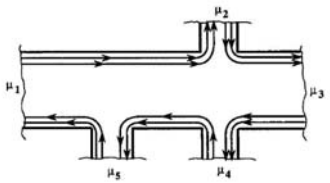


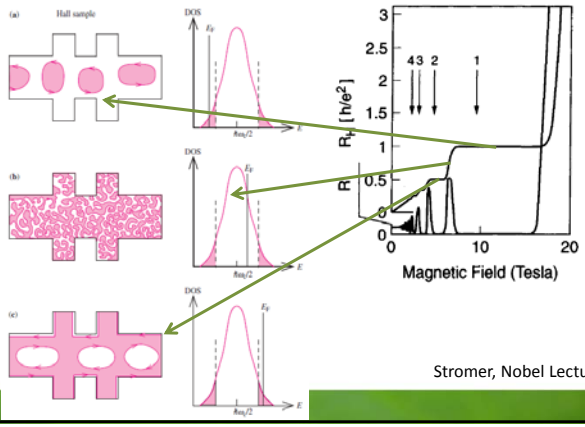
Figure 5.5: Hall bar with five terminals each with chemical potential μ_i. The Fermi energy is set in the Quantum Hall regime such that there are two edge-channels connecting the terminals. Picture from Ferry & Goodnick.

Clive Emary
Theory of Nanostructures nanoskript.pdf

Kwantowy (całkowity) efekt Halla (IQHE)

Integer Quantum Hall Effect (IQHE) – dla gazu 2D jeśli poziom Fermiego znajduje się w obszarze stanów zlokalizowanych opór hallowski jest skwantowany

$$R_H = \frac{1}{\nu}\frac{h}{e^2}$$



Yu, Cardona

Stromer, Nobel Lecture

Kwant … (FQHE)

Fractional Qu … go znajduje się w obszarze stan …

$$R_H = \frac{1}{\nu^*}\frac{h}{e^2}$$

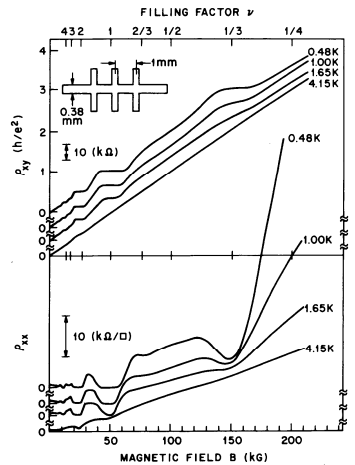


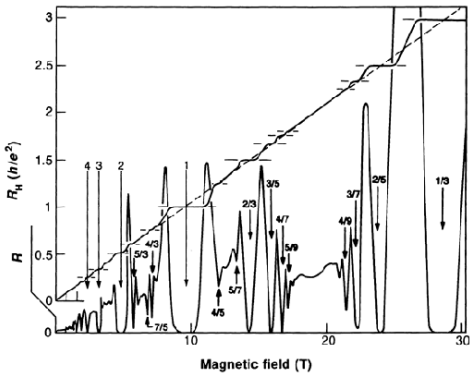
Figure 10. First publication on the FQHE. Hall resistance data (here ρ_{xy}) and magneto resistance data (here ρ_{xx}) are from the same specimen as in Fig. 9. The filling factor, ν, of the Landau level is indicated on the top. The features at ν=1,2,3.. are due to the IQHE. The features at ν=1/3 are due to the FQHE. Sample dimensions and sample temperatures are indicated.

Stromer, Nobel Lecture

Kwantowy ułamkowy efekt Halla (FQHE)

Fractional Quantum Hall Effect (FQHE) – dla gazu 2D jeśli poziom Fermiego znajduje się w obszarze stanów zlokalizowanych opór hallowski jest skwantowany

$$R_H = \frac{1}{\nu}\frac{h}{e^2}$$



http://www.phys.unsw.edu.au/~arh/background/Semiconductor_Devices/semiconductor_devices.html

Kwantowy ułamkowy efekt Halla (FQHE)

Fractional Quantum Hall Effect (FQHE) – dla gazu 2D jeśli poziom Fermiego znajduje się w obszarze stanów zlokalizowanych opór hallowski jest skwantowany

$$R_H = \frac{1}{\nu^*}\frac{h}{e^2}$$

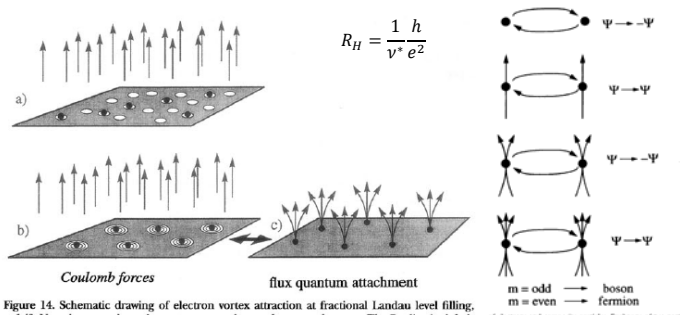


Figure 14. Schematic drawing of electron vortex attraction at fractional Landau level filling, ν=1/3. Now there are three times as many vortices as there are electrons. The Pauli principle is satisfied by placing one vortex onto each electron (a). Placing three vortices onto each electron reduces electron-electron (Coulomb) repulsion (b). Vortex attachment can be viewed as the attachment of magnetic flux quanta to the electrons transforming them to composite particles (c).

Stromer, Nobel Lecture

Composite fermions

Fractional Quantum Hall Effect (FQHE) – composite fermions, fractionally charged quasiparticles

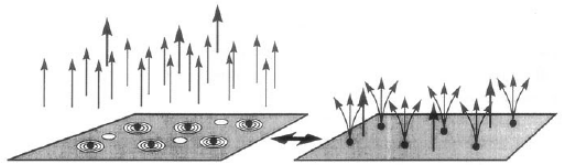


Figure 16. Schematic representation of 1/3 charged quasiparticles. At slightly higher B fields than at ν=1/3 additional vortices are created. They represent dimples in the electron lake. In the dimples exactly 1/3 of an electron charge is missing. These are the fractionally charged quasiparticles of the FQHE.

Stromer, Nobel Lecture

Kropki kwantowe

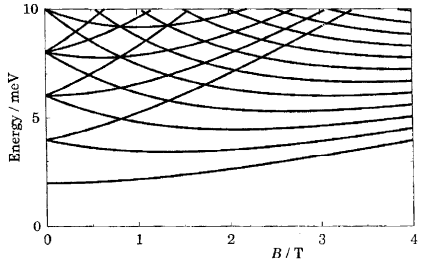


FIGURE 6.16. Energy levels in a magnetic field of a GaAs dot with a parabolic confining potential giving $\hbar\omega_0 = 2$ meV.

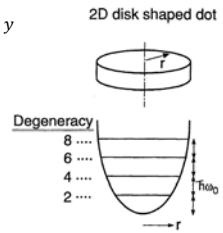
Potencjał harmoniczny 2D

$E_n^x = \hbar\omega_0\left(n_x + \frac{1}{2}\right)$ w kierunku x i taka sama w y

$E_n^y = \hbar\omega_0\left(n_y + \frac{1}{2}\right)$

$E_n = E_n^x + E_n^y = \hbar\omega_0(N+1)$

Degeneracja? $N = n_x + n_y$



Przypomnienie

Fig. 5. Schematic model for the vertical dot with a harmonic lateral potential. The single-particle states are laterally confined into discrete equidistant 0D levels whose degeneracies are 2, 4, 6, 8, ⋯ including spin degeneracy from the lowest level.

Jpn. J. Appl. Phys. Vol. 36 (1997) pp. 3917–3923 Part 1, No. 6B, June 1997

Potencjał harmoniczny 2D

$E_n^x = \hbar\omega_0\left(n_x + \frac{1}{2}\right)$ w kierunku x i taka sama w y

$E_n^y = \hbar\omega_0\left(n_y + \frac{1}{2}\right)$

$E_n = E_n^x + E_n^y = \hbar\omega_0(N+1)$

Degeneracja? $N = n_x + n_y$

$g_N = N + 1$

N	(n_x, n_y)
0	(0,0)
1	(1,0) (0,1)
2	(2,0) (1,1) (0,2)
3	(3,0) (2,1) (1,2) (0,3)

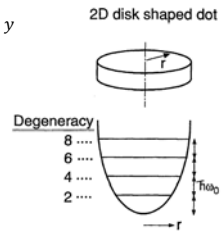
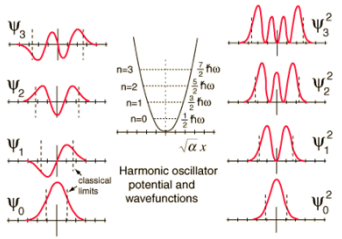


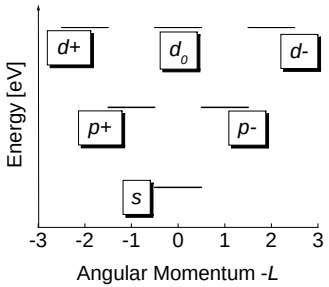
Fig. 5. Schematic model for the vertical dot with a harmonic lateral potential. The single-particle states are laterally confined into discrete equidistant 0D levels whose degeneracies are 2, 4, 6, 8, ⋯ including spin degeneracy from the lowest level.

Jpn. J. Appl. Phys. Vol. 36 (1997) pp. 3917–3923 Part 1, No. 6B, June 1997

Przypomnienie

Potencjał harmoniczny 2D





$n, m = 0,1,2...$

$L = n - m$ (elektron)

Przypomnienie

Adam Babiński

Kropki kwantowe

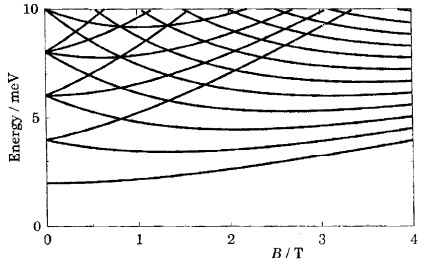


FIGURE 6.16. Energy levels in a magnetic field of a GaAs dot with a parabolic confining potential giving $\hbar\omega_0 = 2$ meV.

Hofstadter butterfly

http://physics.technion.ac.il/~odim/hofstadter.html

The Hofstadter butterfly is the energy spectrum of an electron, restricted to move in two-dimensional periodic potential under the influence of a perpendicular magnetic field. The horizontal axis is the energy and the vertical axis is the magnetic flux through the unit cell of the periodic potential. The flux is a dimensionless number when measured in quantum flux units (will call it ***α***). It is an example of a fractal energy spectrum. When the flux parameter ***α*** is rational and equal to ***p/q*** with ***p*** and ***q*** relatively prime, the spectrum consists of ***q*** non-overlapping energy bands, and therefore ***q+1*** energy gaps (gaps number ***0*** and ***q*** are the regions below and above the spectrum accordingly). When ***α*** is irrational, the spectrum is a cantor set.

De Haas – van Asphen

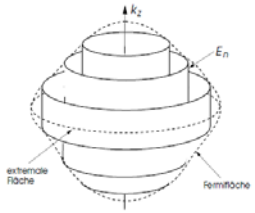
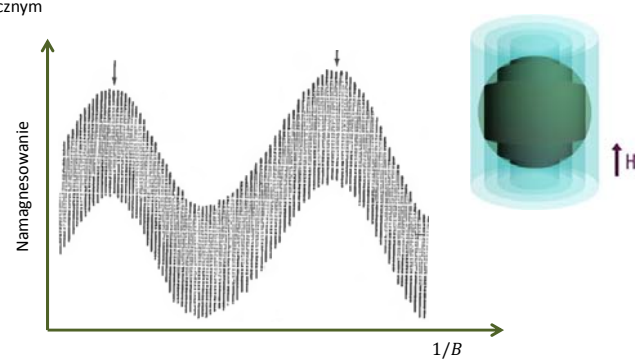


Fig. 3.12: Tubes of quantized electronic states in a magnetic field along the z-axis. A maximum of the magnetization occurs every time a tube crosses the extremal Fermi surface arrea as the magnetic field is increased.

www.itp.phys.ethz.ch/education/lectures_fs10/Solid/Notes.pdf

De Haas – van Asphen

Moment magnetyczny czystego monokryształu metalu oscyluje w zmiennym polu magnetycznym



Ashcroft, Mermin

De Haas – van Asphen

Moment magnetyczny czystego monokryształu metalu oscyluje w zmiennym polu magnetycznym

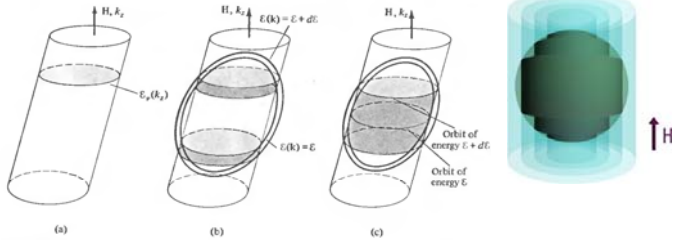


Figure 14.5
(a) A Landau tube. Its cross sections by planes perpendicular to **H** have the same area—$(\nu + \lambda)\,\Delta A$ for the νth tube—and are bounded by curves of constant energy $\varepsilon_\nu(k_z)$ at height k_z. (b) The portion of the tube containing orbits in the energy range from ε to $\varepsilon + d\varepsilon$ when none of the orbits in that range occupy extremal positions on their constant-energy surfaces. (c) Same construction as in (b), except that ε is now the energy of an extremal orbit. Note the great enhancement in the range of k_z for which the tube is contained between the constant-energy surfaces at ε and $\varepsilon + d\varepsilon$.

Ashcroft, Mermin

De Haas – van Asphen

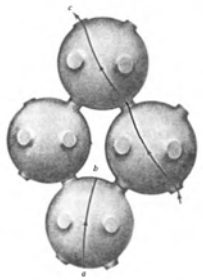


Figure 15.7
Indicating only a few of the surprisingly many types of orbits an electron can pursue in k-space when a uniform magnetic field is applied to a noble metal. (Recall that the orbits are given by slicing the Fermi surface with planes perpendicular to the field.) The figure displays (a) a closed particle orbit; (b) a closed hole orbit; (c) an open orbit, which continues in the same general direction indefinitely in the repeated-zone scheme.

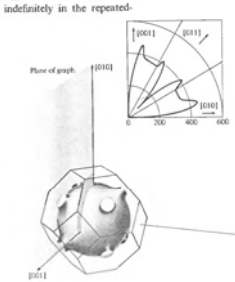


Figure 15.8
The spectacular direction dependence of the high-field magnetoresistance in copper that is characteristic of a Fermi surface supporting open orbits. The [001] and [010] directions of the copper crystal are as indicated in the figure, and the current flows in the [100] direction perpendicular to the graph. The magnetic field is in the plane of the graph. Its magnitude is fixed at 18 kilogauss, and its direction varied continuously from [001] to [010]. The graph is a polar plot of

$$\frac{\rho(H) - \rho(0)}{\rho(0)}$$

vs. orientation of the field. The sample is very pure and the temperature very low (4.2 K—the temperature of liquid helium) to insure the highest possible value for $\omega_c\tau$. (J. R. Klauder and J. E. Kunzler, *The Fermi Surface*, Harrison and Webb, Wiley, New York, 1960.)

Ashcroft, Mermin

De Haas – van Asphen

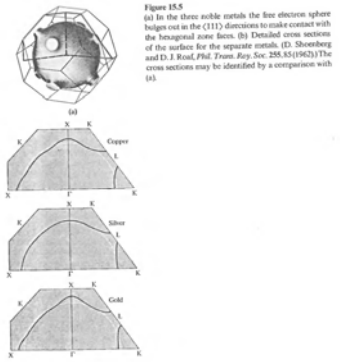


Figure 15.5
(a) In the three noble metals the free electron sphere bulges out in the ⟨111⟩ directions to make contact with the hexagonal zone faces. (b) Detailed cross sections of the surface for the separate metals. (D. Shoenberg and D. J. Roaf, *Phil. Trans. Roy. Soc.* 255.85 (1962).) The cross sections may be identified by a comparison with (a).

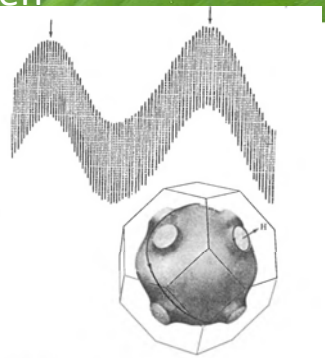


Figure 15.6
De Haas-van Alphen oscillations in silver. (Courtesy of A. S. Joseph.) The magnetic field is along a ⟨111⟩ direction. The two distinct periods are due to the neck and belly orbits indicated in the inset, the high-frequency oscillations coming from the larger belly orbit. By merely counting the number of high-frequency periods in a single low-frequency period (i.e., between the two arrows) one deduces directly that $A_{111}(\text{belly})/A_{111}(\text{neck}) = 51$. (Note that it is not necessary to know either the vertical or horizontal scales of the graph to determine this fundamental piece of geometrical information!)

Ashcroft, Mermin

fot. AcidCow.com