TIME IN QUANTUM COSMOLOGY

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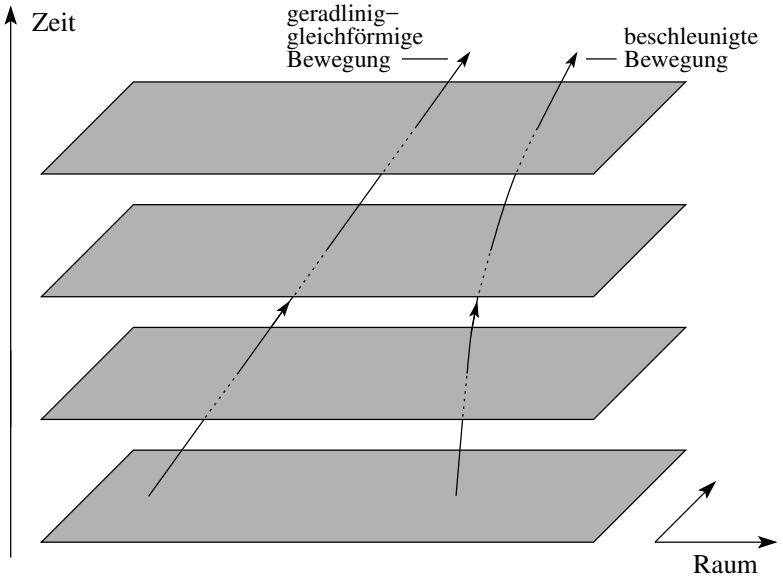
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The direction of time

Absolute time and space



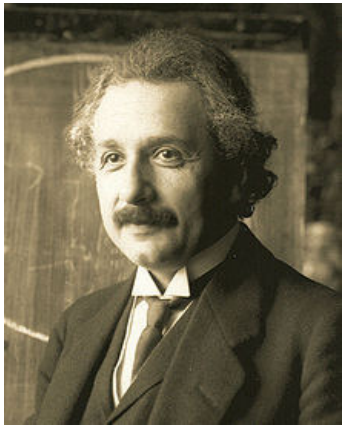
Isaac Newton 1687:

I. Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration. . . .

II. Absolute space, in its own nature, without relation to anything external, remains always similar and immovable. . . .

Einsteinian gravity

- General theory of relativity



$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

(November 25, 1915)

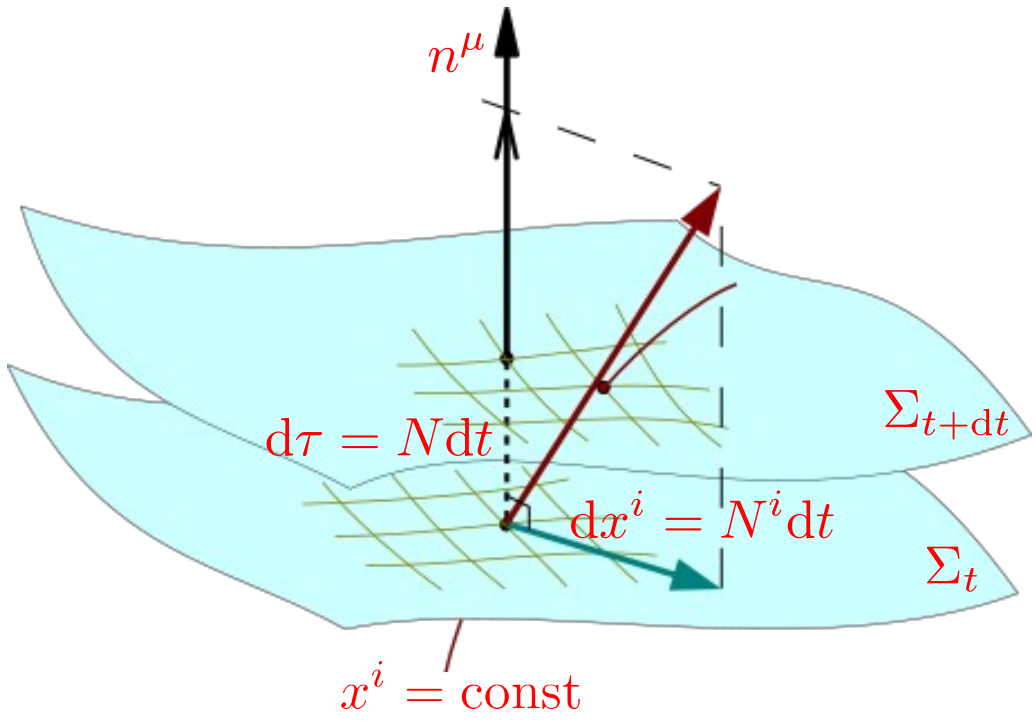
- left: geometry (gravitational field)
- right: matter fields (Standard model of particle physics)

Background independence

Albert Einstein:
Es widerstrebt dem wissenschaftlichen Verstande, ein Ding zu setzen, das zwar wirkt, aber auf das nicht gewirkt werden kann.

(It is contrary to the scientific mode of understanding to postulate a thing that acts, but which cannot be acted upon.)

There are no absolute fields in general relativity, that is, no background structure.



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General relativity in canonical form

Einstein's equations can be written as a dynamical system (for the three-metric h_{ab} and its canonical momentum π^{ab} on a spacelike hypersurface Σ) of evolution equations together with constraints:

$$\begin{aligned}\mathcal{H}_\perp &= 2\kappa\, G_{ab\,cd}\pi^{ab}\pi^{cd} - (2\kappa)^{-1}\sqrt{h}({}^{(3)}R - 2\Lambda) + \sqrt{h}\rho \approx 0\\ \mathcal{H}^a &= -2\nabla_b\pi^{ab} + \sqrt{h}j^a \approx 0\,,\end{aligned}$$

with the DeWitt metric

$$G_{ab\,cd} = \tfrac{1}{2\sqrt{h}}(h_{ac}h_{bd} + h_{ad}h_{bc} - h_{ab}h_{cd})$$

and

$$\kappa = 8\pi G/c^4$$

$H \approx 0$ is called "Hamiltonian constraint", $\mathcal{H}^a \approx 0$ are called "momentum (diffeomorphism) constraints".

Constraints and evolution

I

Constraints are preserved in time $\Longleftrightarrow$ energy–momentum tensor of matter has vanishing covariant divergence

compare with electrodynamics: Gauss constraint preserved in time $\Longleftrightarrow$ charge conservation

II

Einstein's equations are the unique propagation law consistent with the constraints

compare with electrodynamics: Maxwell's equations are the unique propagation law consistent with the Gauss constraint

Problem of time I

Restriction to *compact* three-spaces Σ:

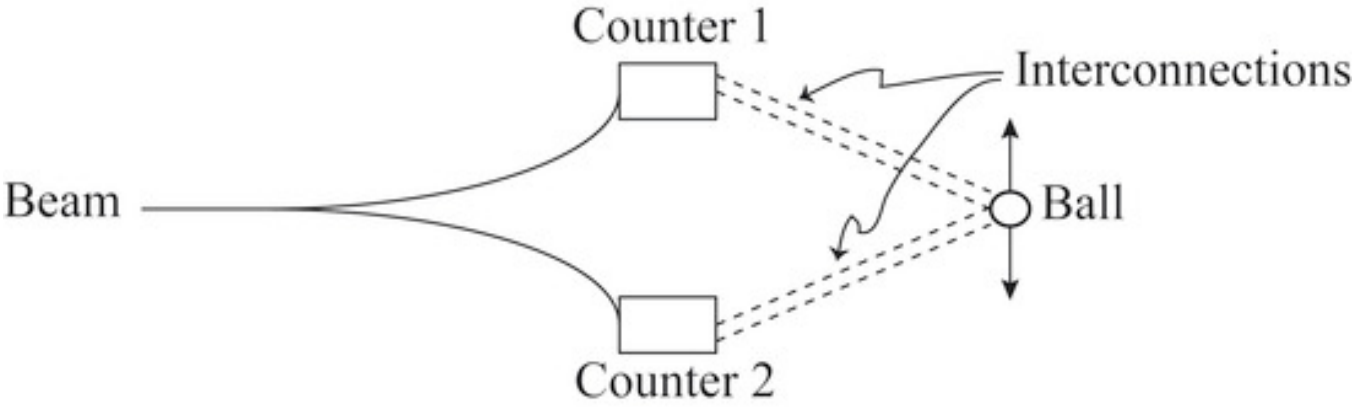
- The total Hamiltonian is a combination of pure constraints; all of the evolution will be generated by constraints;
- no external time parameter exists
- all physical time parameters are to be constructed from within our system, that is, as functional of the canonical variables; a priori there is no preferred choice of such an intrinsic time parameter
- Observables? Commute with a gauge generator that is a tuned *sum* of all constraints

The absence of an extrinsic time and the non-preference of an intrinsic one is known as the problem of time in (classical) canonical gravity. Still, spacetime exists.

Quantum Gravity

Richard Feynman 1957:

. . . if you believe in quantum mechanics up to any level then you have to believe in gravitational quantization in order to describe this experiment. . . . It may turn out, since we've never done an experiment at this level, that it's not possible . . . that there is something the matter with our quantum mechanics when we have too much *action* in the system, or too much mass—or something. But that is the only way I can see which would keep you from the necessity of quantizing the gravitational field. It's a way that I don't want to propose. . . .



Main approaches to quantum gravity

No question about quantum gravity is more difficult than the question, "What is the question?"
(John Wheeler 1984)

- Quantum general relativity
  - Covariant approaches (perturbation theory, path integrals including spin foams, asymptotic safety, . . .)
  - Canonical approaches (geometrodynamics, connection dynamics, loop dynamics, . . .)
- String theory
- Fundamental discrete approaches
  (quantum topology, causal sets, group field theory, . . .);
  have partially grown out of the other approaches

Planck units

$$
\begin{aligned}
l_{\mathrm{P}} &= \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-35}\ \mathrm{m} \\
t_{\mathrm{P}} &= \frac{l_{\mathrm{P}}}{c} = \sqrt{\frac{\hbar G}{c^5}} \approx 5.40 \times 10^{-44}\ \mathrm{s} \\
m_{\mathrm{P}} &= \frac{\hbar}{l_{\mathrm{P}} c} = \sqrt{\frac{\hbar c}{G}} \approx 2.17 \times 10^{-8}\ \mathrm{kg} \approx 1.22 \times 10^{19}\ \mathrm{GeV}/c^2
\end{aligned}
$$

Max Planck (1899):

Diese Grössen behalten ihre natürliche Bedeutung so lange bei, als die Gesetze der Gravitation, der Lichtfortpflanzung im Vacuum . . . in Gültigkeit bleiben, sie müssen also, von den verschiedensten Intelligenzen nach den verschiedensten Methoden gemessen, sich immer wieder als die nämlichen ergeben.

Canonical quantum gravity

Central equations are constraints:

$$\boxed{\hat{H}\Psi = 0}$$

Different canonical approaches:

- Geometrodynamics –
  metric and extrinsic curvature
- Connection dynamics –
  connection (A_a^i) and coloured electric field (E_i^a)
- Loop dynamics –
  flux of E_i^a and holonomy

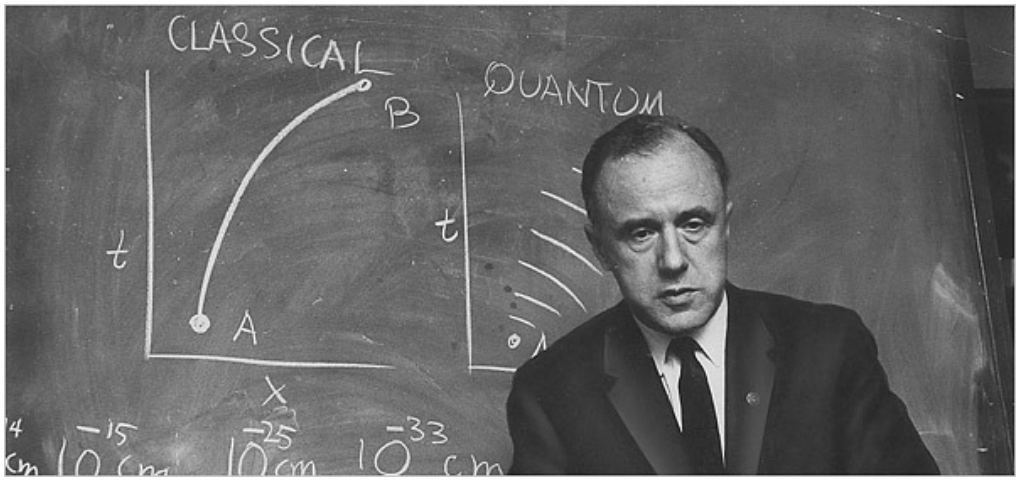
Erwin Schrödinger 1926:

We know today, in fact, that our classical mechanics fails for very small dimensions of the path and for very great curvatures. Perhaps this failure is in strict analogy with the failure of geometrical optics . . . that becomes evident as soon as the obstacles or apertures are no longer great compared with the real, finite, wavelength. . . . Then it becomes a question of searching for an undulatory mechanics, and the most obvious way is by an elaboration of the Hamiltonian analogy on the lines of undulatory optics.[1]

---

[1] *wir wissen doch heute, daß unsere klassische Mechanik bei sehr kleinen Bahndimensionen und sehr starken Bahnkrümmungen versagt*. Vielleicht ist dieses Versagen eine volle Analogie zum Versagen der geometrischen Optik . . . , das bekanntlich eintritt, sobald die 'Hindernisse' oder 'Öffnungen' nicht mehr groß sind gegen die wirkliche, endliche Wellenlänge. . . . Dann gilt es, eine 'undulatorische Mechanik' zu suchen – und der nächstliegende Weg dazu ist wohl die wellentheoretische Ausgestaltung des Hamiltonschen Bildes.

Quantum geometrodynamics



(a) John Archibald Wheeler



(b) Bryce DeWitt

Application of Schrödinger's procedure to general relativity leads to

$$\hat{\mathcal{H}}_{\perp}\Psi \equiv \left(-16\pi G\hbar^2 G_{abcd}\frac{\delta^2}{\delta h_{ab}\delta h_{cd}} - (16\pi G)^{-1}\sqrt{h}\left({}^{(3)}R - 2\Lambda\right) + \sqrt{h}\hat{\rho}\right)\Psi = 0$$

Wheeler–DeWitt equation

$$\hat{\mathcal{H}}^a\Psi \equiv -2\nabla_b\frac{\hbar}{\mathrm{i}}\frac{\delta\Psi}{\delta h_{ab}} + \sqrt{h}\hat{j}^a\Psi = 0$$

quantum diffeomorphism (momentum) constraint

Comparison between geometrodynamics and particle dynamics		
underlying notion	geometrodynamics	particle dynamics
dynamical object	space	particle
configuration	h_{ij} 3-geometry	x, t events
classical description	$h_{ij}(t)$ 4-geometry	$x(t)$ trajectory
dynamical arena	superspace	spacetime
wave function(al)	$\Psi[h_{ij}]$	$\psi(x, t)$

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Problem of time II

- External time t has vanished from the formalism.
- This holds also for loop quantum gravity and probably for string theory;
- it holds for *any* theory that has no absolute time at the classical level.
- The Wheeler–DeWitt equation has the structure of a wave equation any may therefore allow the introduction of an intrinsic time.
- The Hilbert-space structure in quantum mechanics is connected with the probability interpretation, in particular with probability conservation *in time* t; what happens with this structure in a timeless situation?

- Concepts also apply to the path-integral approach: the quantum-gravitational path integral is formally solved by the quantum constraints
- The problem of time is similar in loop quantum gravity, but somewhat different in string theory (which is a theory of emergent gravity)
- Remarks on observables

Quantum Cosmology

Closed Friedmann–Lemaître universe with scale factor a, containing a homogeneous massive scalar field ϕ (two-dimensional *minisuperspace*)

$$\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\mathrm{d}\Omega_3^2$$

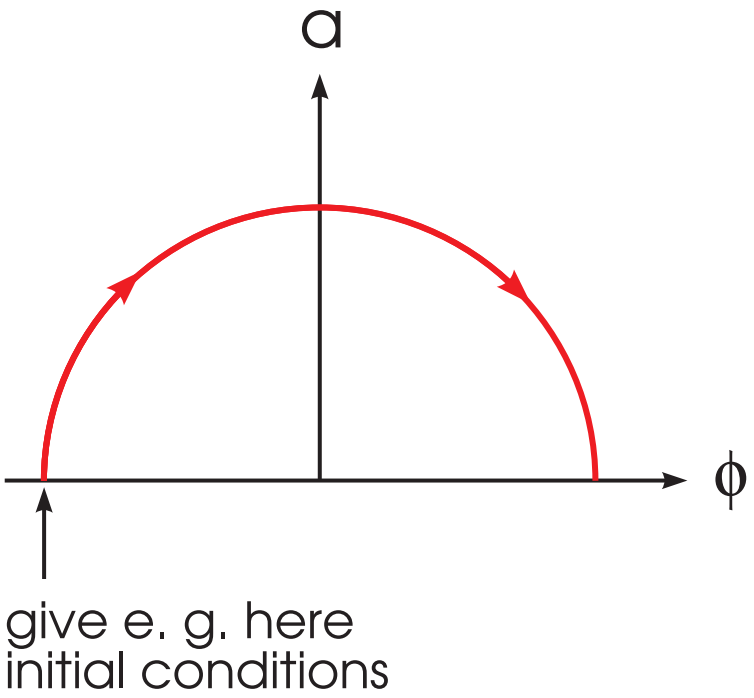
The Wheeler–DeWitt equation reads (with units $2G/3\pi = 1$)

$$\frac{1}{2}\left(\frac{\hbar^2}{a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) - \frac{\hbar^2}{a^3}\frac{\partial^2}{\partial\phi^2} - a + \frac{\Lambda a^3}{3} + m^2a^3\phi^2\right)\psi(a,\phi) = 0$$

Factor ordering chosen in order to achieve covariance in minisuperspace

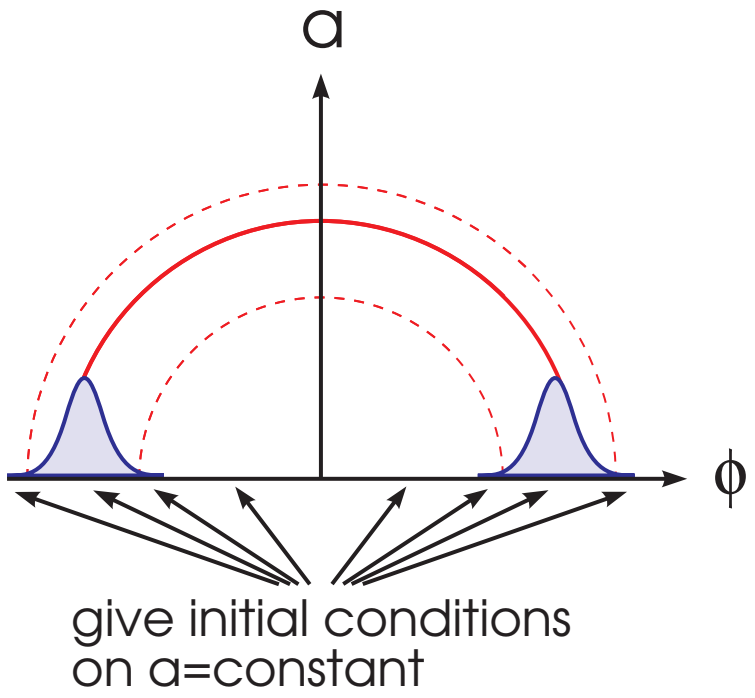
Determinism in classical and quantum theory

Classical theory



Recollapsing part is deterministic successor of expanding part

Quantum theory



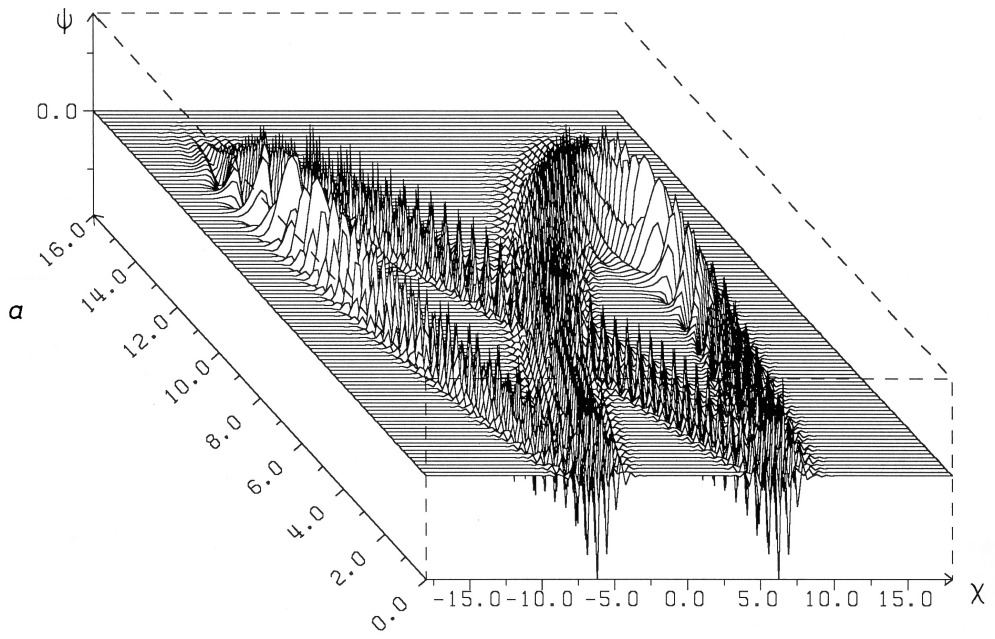
‘Recollapsing’ wave packet must be present ‘initially’

No intrinsic difference between ‘big bang’ and ‘big crunch’!

Example

Indefinite Oscillator

$$\hat{H}\psi(a,\chi) \equiv (-H_a + H_\chi)\psi \equiv \left(\frac{\partial^2}{\partial a^2} - \frac{\partial^2}{\partial \chi^2} - a^2 + \chi^2\right)\psi = 0$$



C.K. (1990)

Recovery of time

An expansion of the Wheeler–DeWitt equation with respect to the Planck mass leads to the functional Schrödinger equation for non-gravitational fields (with Hamiltonian $\hat{H}^{\mathrm{m}}$) in a spacetime that is a solution of Einstein's equations. In this way, a semiclassical (WKB) time emerges as an approximate concept.
(Born–Oppenheimer type of approximation)

In this limit, one recovers the usual Hilbert space structure and the associated probability interpretation.

Lapchinsky and Rubakov 1979, *Banks* 1985, *Halliwell and Hawking* 1985, *Hartle* 1986, *C.K.* 1987, ...

Quantum-gravitational corrections

The next order in the Born–Oppenheimer approximation gives

$$\hat{H}^{\mathrm{m}} \to \hat{H}^{\mathrm{m}} + \frac{1}{m_{\mathrm{P}}^2}\,(\text{various terms})$$

C.K. and Singh (1991); *Barvinsky and C.K.* (1998)

- Quantum-gravitational correction to energy values
- Possible contribution to the CMB anisotropy spectrum

Observations

Does the anisotropy spectrum of the cosmic background radiation contain information about quantum gravity?

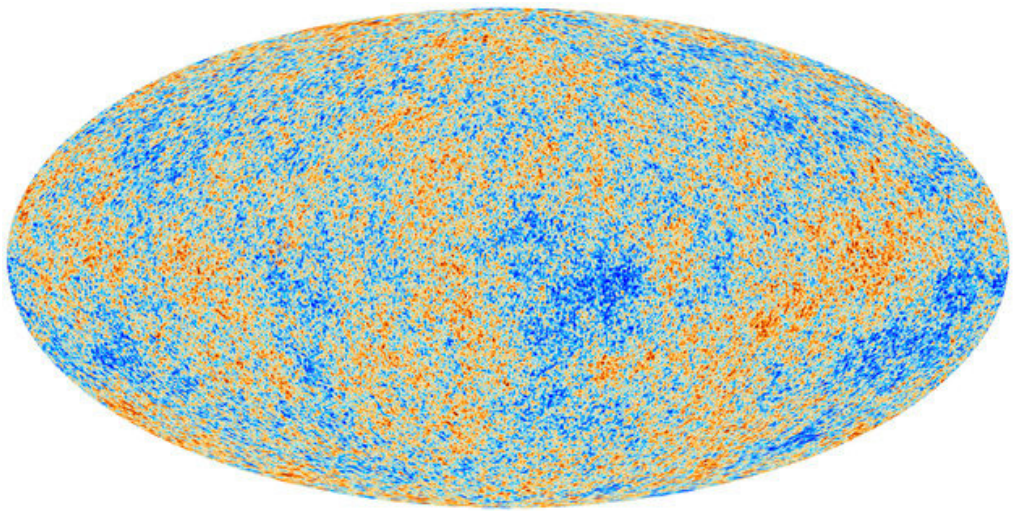


Figure credit: ESA/PLANCK Collaboration

C.K. and M. Krämer, *Phys. Rev. Lett.*, **108**, 021301 (2012); D. Bini, G. Esposito, C.K., M. Krämer, and F. Pessina, *Phys. Rev. D*, **87**, 104008 (2013); D. Brizuela, C.K., M. Krämer, *ibid.* **93**, 104035 (2016); *ibid.* **94**, 123527 (2016); D. Brizuela, C.K., M. Krämer, S. Robles-Pérez, *ibid.* **99**, 104007 (2019); L. Chataignier and M. Krämer, *ibid.* **103**, 066005 (2021).

Decoherence in quantum cosmology

In quantum cosmology, arbitrary superpositions of the gravitational field and matter states can occur. How can we understand the emergence of an (approximate) classical Universe?

Introduction of inhomogeneities

Describe small inhomogeneities by multipoles $\{x_n\}$ around the minisuperspace variables (e.g. a and ϕ)

$$\left(H_0 + \sum_n H_n(a, \phi, x_n) \right) \Psi(\alpha, \phi, \{x_n\}) = 0$$

(Halliwell and Hawking 1985)

If ψ_0 is of WKB form, $\psi_0 \approx C \exp(\mathrm{i}S_0/\hbar)$ (with a slowly varying prefactor C), one gets with $\Psi = \psi_0 \prod_n \psi_n$,

$$\mathrm{i}\hbar \frac{\partial \psi_n}{\partial t} \approx H_n \psi_n$$

with

$$\frac{\partial}{\partial t} \equiv \nabla S_0 \cdot \nabla$$

t: 'WKB time' – controls the dynamics in this approximation

Decoherence in quantum cosmology

- 'System': global degrees of freedom (scale factor, inflaton field, ...)
- 'Environment': small density fluctuations, gravitational waves, ...

(Zeh 1986, C.K. 1987)

Example: scale factor a of a de Sitter universe ($a \propto \mathrm{e}^{H_{\mathrm{I}} t}$) ('system') experiences decoherence by gravitons ('environment') according to

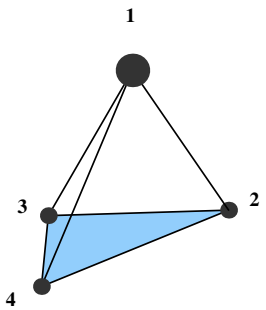
$$\rho_0(a, a') \to \rho_0(a, a') \exp\left(-C H_{\mathrm{I}}^3 a (a - a')^2\right) \ , \ C > 0$$

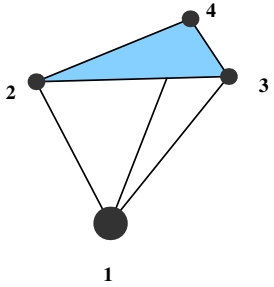
The Universe assumes classical properties at the beginning of inflation

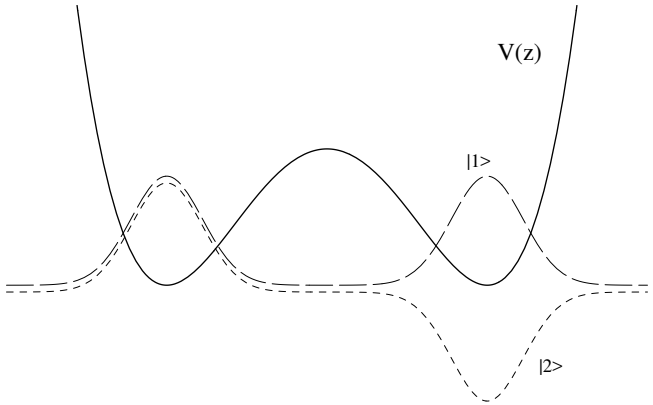
(Barvinsky, Kamenshchik, C.K. 1999)

Time from symmetry breaking

Analogy from molecular physics: emergence of chirality







dynamical origin: decoherence through scattering by light or air molecules

Quantum cosmology: decoherence between $\exp(\mathrm{i}S_0/G\hbar)$- and $\exp(-\mathrm{i}S_0/G\hbar)$-components of the wave function through interaction with e.g. weak gravitational waves

Example for decoherence factor:
$\exp\left(-\frac{\pi m H_0^2 a^3}{128\hbar}\right) \sim \exp\left(-10^{43}\right)$ (C.K. 1992)

Decoherence of primordial fluctuations

During the inflationary phase (ca. 10^{-34} after the Big Bang) there is a quantum-to-classical transition for the ubiquitous fluctuations of the inflaton and the metric.
The process of decoherence is crucial in understanding this transition

The fluctuations then behave like classical stochastic quantities and yield the seeds for the structures in the Universe. Quantum gravity is needed to understand the power spectrum.

(C.K., Polarski, Starobinsky 1998, . . .)



Interpretation of quantum cosmology

Both quantum general relativity and string theory preserve the linear structure for the quantum states;
strict validity of the superposition principle

B. S. DeWitt 1967:

Everett's view of the world is a very natural one to adopt in the quantum theory of gravity, where one is accustomed to speak without embarassment of the 'wave function of the universe.' It is possible that Everett's view is not only natural but essential.

- Remarks on interpretations other than Everett (de Broglie–Bohm, collapse, . . .)
- Remarks on *reduced* approaches and on singularity avoidance[2]

[2] E.g. P. Małkiewicz, P. Peter, S. Vitenti, PRD **101**, 046012 (2020); W. Piechocki, arXiv:2006.05242v2; M. Bouhmadi-López, C.K., P. Martín-Moruno, GRG **51**, 10 (2019), . . .

The direction of time



Paul Cézanne, *Nature morte au crâne* (Barnes Foundation, Pennsylvania)

Arrow of time from cosmology

Where does the Sun come from?

$\Downarrow$

Gravitational instability of dust clouds

$\Downarrow$

Cosmology

Ludwig Boltzmann (1898):

That in Nature the transition from a probable to an improbable state does not happen equally often as the opposite transition, should be sufficiently explained by the assumption of a very improbable initial state of the whole Universe surrounding us . . . .

How special is the Universe?

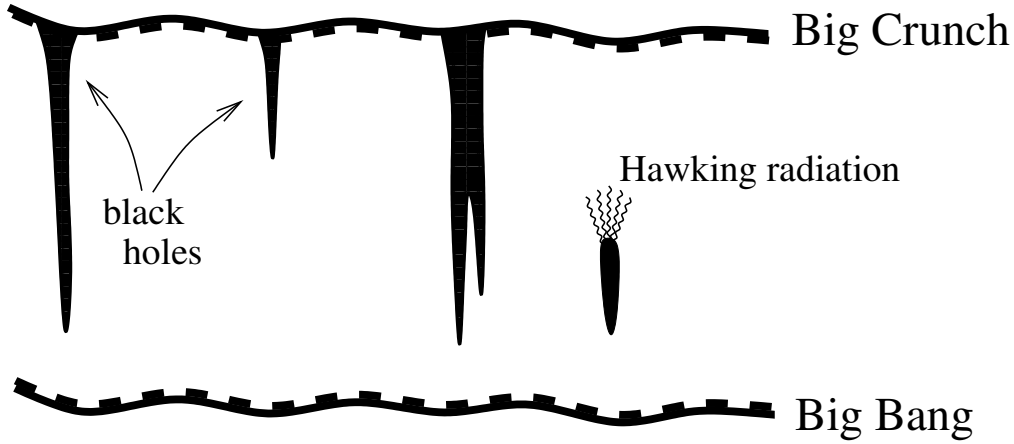
Penrose (1981):

Entropy of the observed part of the Universe is maximal if all its mass is in one black hole; the probability for our Universe would then be (updated version from C.K. arXiv:0910.5836)

$$\frac{\exp\left(\frac{S}{k_{\mathrm{B}}}\right)}{\exp\left(\frac{S_{\mathrm{max}}}{k_{\mathrm{B}}}\right)} \sim \frac{\exp\left(3.1 \times 10^{104}\right)}{\exp\left(1.8 \times 10^{121}\right)} \approx \exp\left(-1.8 \times 10^{121}\right)$$

Weyl curvature hypothesis

HYPOTHESIS (CLASSICAL)*:* The Weyl curvature vanishes at all past singularities, as the singularity is approached from future directions. *(This condition can be weakened by only demanding that the Weyl tensor be finite, rather than diverging)*



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Quantum Weyl curvature hypothesis

HYPOTHESIS (QUANTUM)*:* The quantum states for the Weyl scalars describing scalar and tensor modes assume the form of adiabatic vacuum states in a (quasi-) de Sitter space, as the region of small-enough scale factors is approached from future directions.

C.K., *AVS Quantum Science* **4**, 015607 (2022).

Arrow of time from quantum cosmology

Fundamental asymmetry with respect to “intrinsic time”:

$$\hat{H}\Psi = \left(\frac{\partial^2}{\partial \alpha^2} + \sum_i \left[-\frac{\partial^2}{\partial x_i^2} + \underbrace{V_i(\alpha, x_i)}_{\to 0 \text{ for } \alpha \to -\infty} \right] \right) \Psi = 0$$

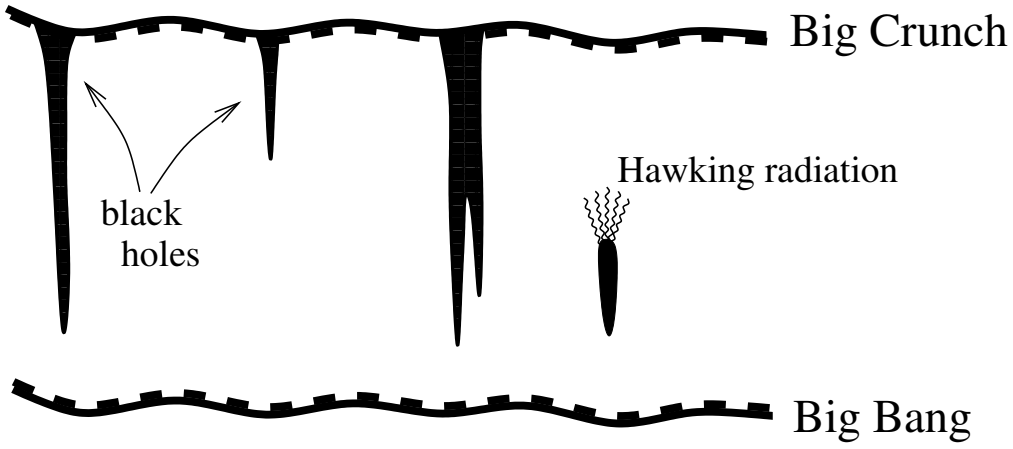
Is compatible with simple boundary condition:

$$\Psi \overset{\alpha \to -\infty}{\longrightarrow} \psi_0(\alpha) \prod_i \psi_i(x_i)$$

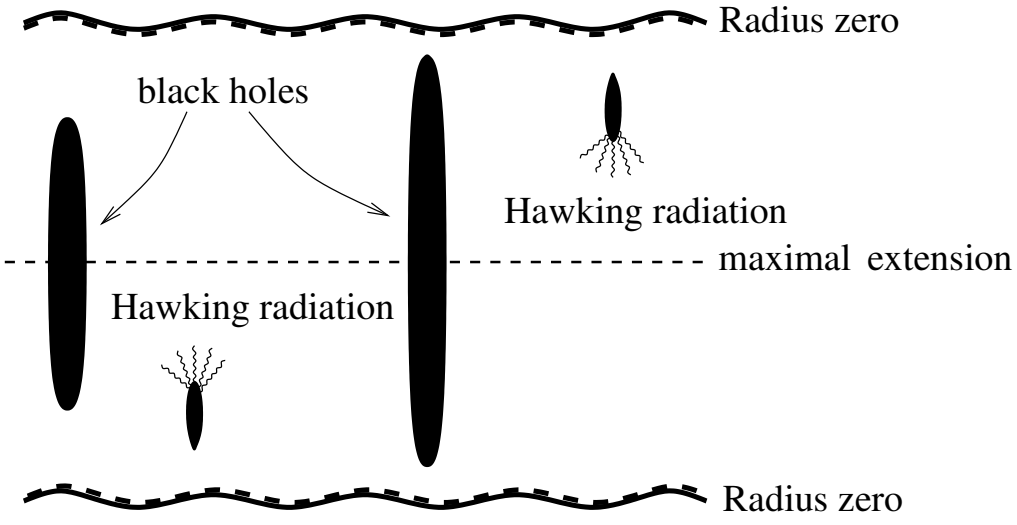
Entropy increases with increasing α, since entanglement with other degrees of freedom increases;
this defines the direction of time

Is the expansion of the Universe a tautology?

Arrow of time in a recollapsing quantum universe



(Penrose 1979)



(C.K. and Zeh 1995)

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John Wheeler (1968):

These considerations reveal that the concepts of spacetime and time itself are not primary but secondary ideas in the structure of physical theory. These concepts are valid in the classical approximation. However, they have neither meaning nor application under circumstances when quantum-geometrodynamical effects become important. . . . There is no spacetime, there is no time, there is no before, there is no after. The question what happens “next” is without meaning.

Bryce DeWitt (1999):

. . . one learns that time and probability are both *phenomenological* concepts.

Literature

- C.K., *Quantum Gravity*, third edition (Oxford University Press, Oxford, 2012);

- C.K. and P. Peter, Time in Quantum Cosmology, *Universe* **2022**, *8*, 36.

- C.K., *Der Quantenkosmos* (S. Fischer, Frankfurt am Main, 2008); Polish translation: *Kwantowy Kosmos* (Copernicus Center Press, Kraków, 2018).

  Kwantowa kosmologia – to tam, gdzie fizyka rodzi się z metafizyki. (Michał Heller)