



Calculators of two-body kinematics

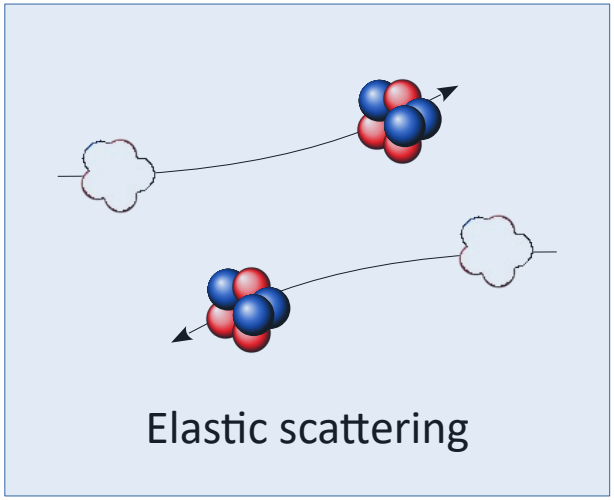
Krzysztof Piasecki

[NRV] I. Zagrebaev et al. , JINR Dubna
[LISE++] O.B Tarasov, D. Bazin , MSU East Lansing

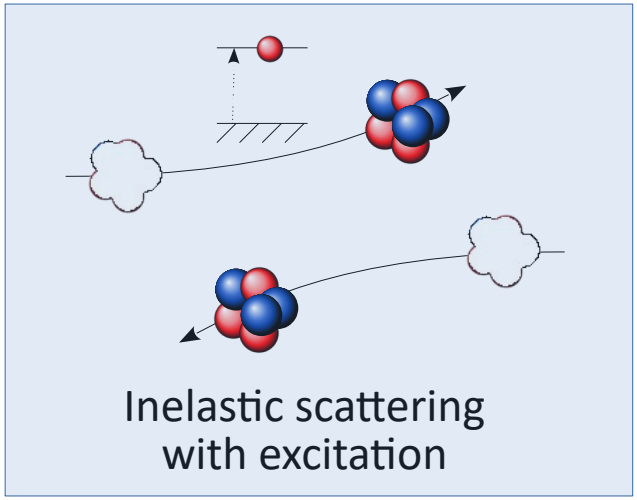
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Scattering and reactions of two nuclei (lower energies)

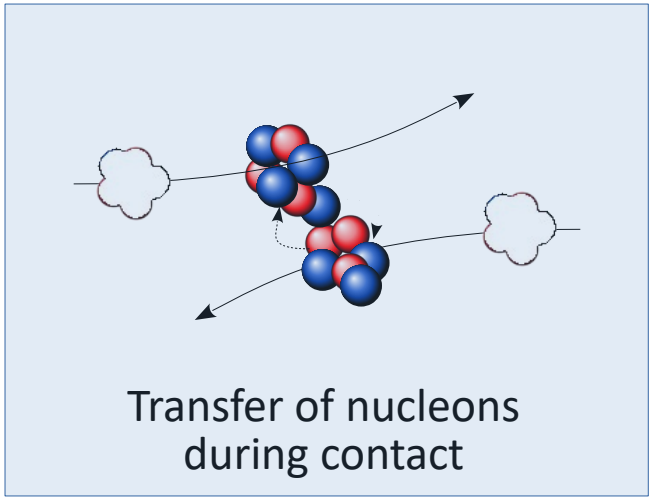
- Two nuclei in the outgoing channel



Elastic scattering

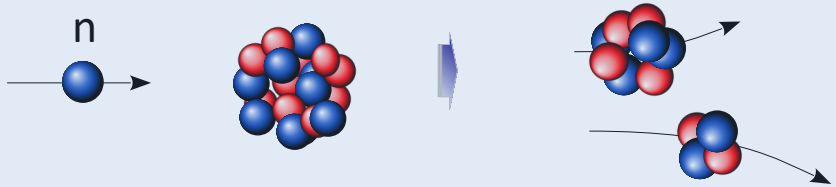


Inelastic scattering
with excitation

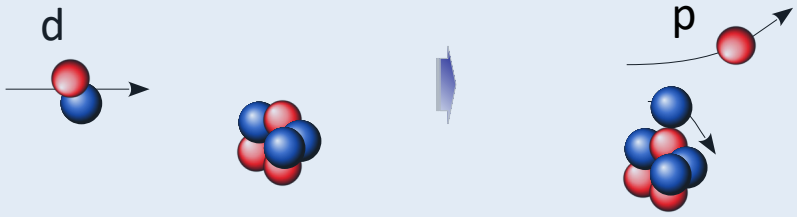


Transfer of nucleons
during contact

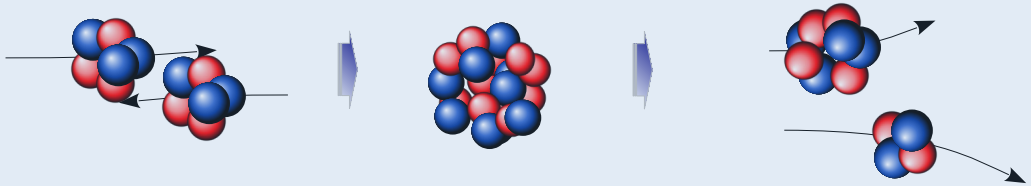
- Two / more nuclei in the outgoing channel + possible further emission of γ / n



Fission into fragments

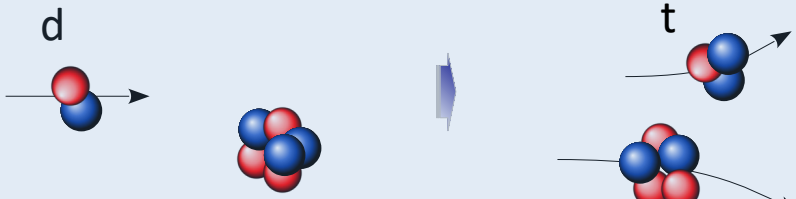


Breakup / Stripping



Fusion into
Compound nucleus

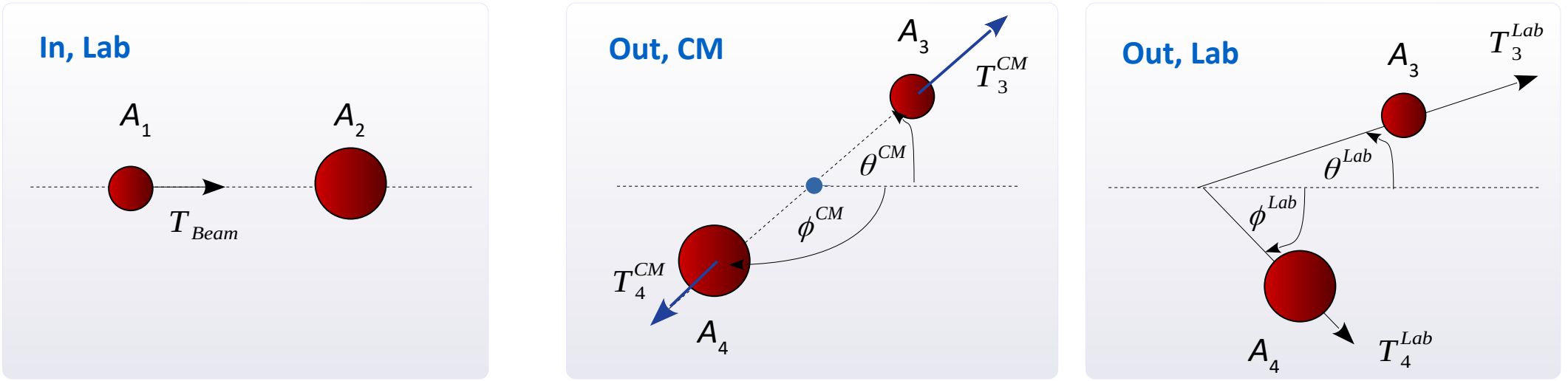
Possibly:
Fission into fragments



Pickup

Some facts on non-relativistic kinematics of elastic collision

- The beam nucleus A_1 is accelerated to kinetic energy T_{Beam} and hits the stationary target nucleus A_2.



- If the scattering or collision is elastic, $A_3 = A_1$ and $A_2 = A_4$. Applying *E* and *p* conservation laws gives:

$$T_3^{Lab} = T_{Beam}\,\frac{\left(A_1 + A_2\cos\theta^{CM}\right)^2 + \left(A_2\sin\theta^{CM}\right)^2}{\left(A_1 + A_2\right)^2}$$

where

$$\tan\theta^{Lab} = \frac{\sin\theta^{CM}}{\cos\theta^{CM} + \dfrac{A_1}{A_2}}$$

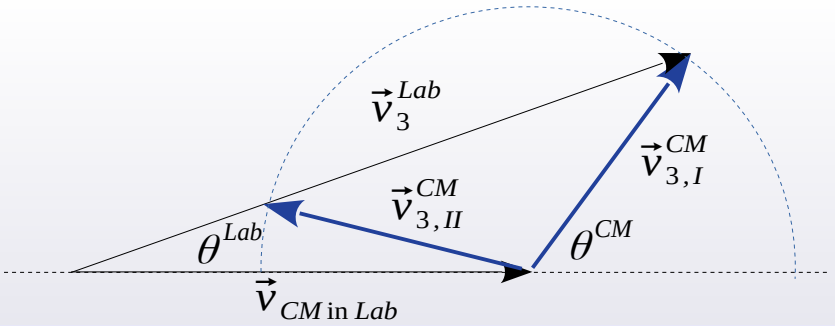
$$T_4^{Lab} = T_{Beam}\,\frac{A_1\,A_2}{\left(A_1 + A_2\right)^2}\left(2\sin\phi^{Lab}\right)^2$$

$$\phi^{Lab} = \frac{\pi}{2} - \frac{\phi^{CM}}{2}$$

- ***Caution***: if $A_1 > A_2$, then:

1) one θ^{Lab} is realized by two θ^{CM}
2) θ^{Lab} has an upper limit.

This situation is called **reverse kinematics**.



Heat of reaction Q

- **Energy balance** for a two-body process (not necessarily elastic one), in the CM frame:

Input channel			=	Output channel					
$m_1 c^2$	+ $m_2 c^2$	+ T_{Beam}		$m_3 c^2$	+ $m_4 c^2$	+ T_3^{CM}	+ T_4^{CM}	+ ϵ_3^*	+ ϵ_4^*
Masses of input nuclei		Kin. energy of beam nucleus		Masses of output nuclei		Kin. energies of output nuclei		Excitation energies of output nuclei	

- Imagine a specific output channel (nuclei of masses m_3, m_4 in excited states of energies ϵ_3^* , ϵ_4^*).
Can this reaction undergo freely? Does it release (kinetic) energy spontaneously?
Or should energy be pumped (via beam kinetic energy) ?

- **Heat of reaction, Q:**

$$Q \overset{df}{=} (m_1 + m_2) - (m_3 + m_4) - (\epsilon_3^* + \epsilon_4^*)$$

$$\Rightarrow \quad Q \overset{numerically}{=} T_3^{CM} + T_4^{CM} - T_{Beam}$$

If $Q \geq 0$, this process can occur spontaneously

If $Q < 0$, it cannot. We must supply this amount of energy (via beam kinetic energy)

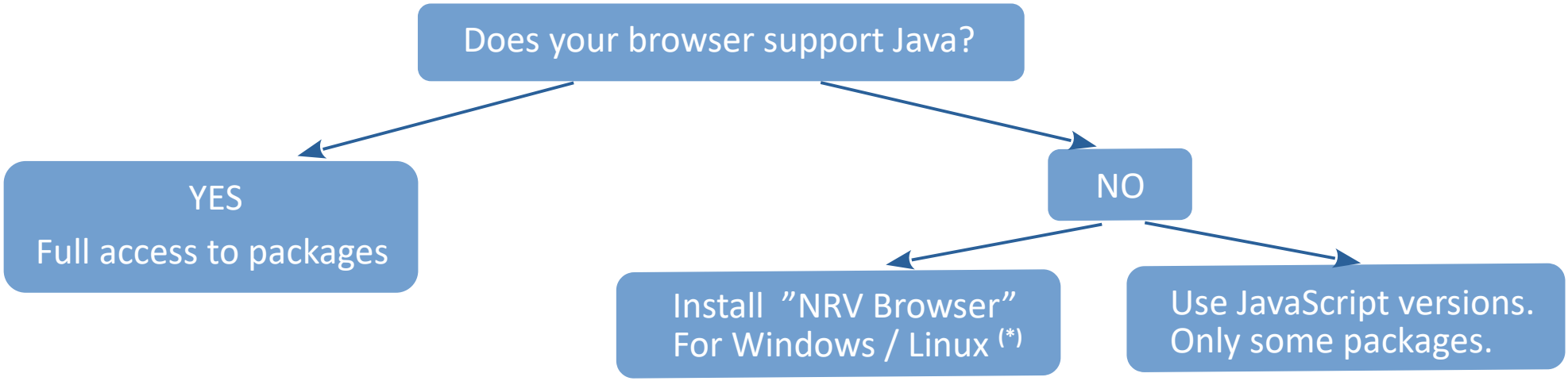
If the collision is elastic, then $Q = 0$.

Package for nuclear data and low-energy collisions

- **Nuclear Reactions Video** by V. Zagrebaev's group @ JINR Dubna around 1999.

  It is a "low-energy knowledge base" with nuclide chart, data on nuclei, nuclear processes, kinematics calculator.

  Flagship paper: A.V. Karpov *et al.*, "NRV web of knowledge base on low-energy nuclear physics", Nuclear Instruments and Methods A 859, 112 (2017)



- **Nuclear properties** – chart of nuclides. Data for each nuclide: binding energies, deformations, excited states (E, spins) and decay properties (BR, Q-values).

- **Systematics** – provide graphs of: separation / binding energy, size, half-life, deformation, fission barriers as function of A / Z / N for a selected one of these values.

- **Kinematics calculator** – provides translations between energies and emission angles of nuclei emitted in 2-body nuclear collisions:
  - elastic scattering (from Coulomb or nuclear interaction)
  - inelastic scattering (with excitation of any of nuclei; via Coulomb or nuclear forces)
  - transfer of some nucleons between nuclei during the collision

(*) On NPD training computer, type: ① `cp -r ~kpiasecki/soft/nrv/ctnp/* .` ② `./run.sh`

Two-body kinematics: user interface

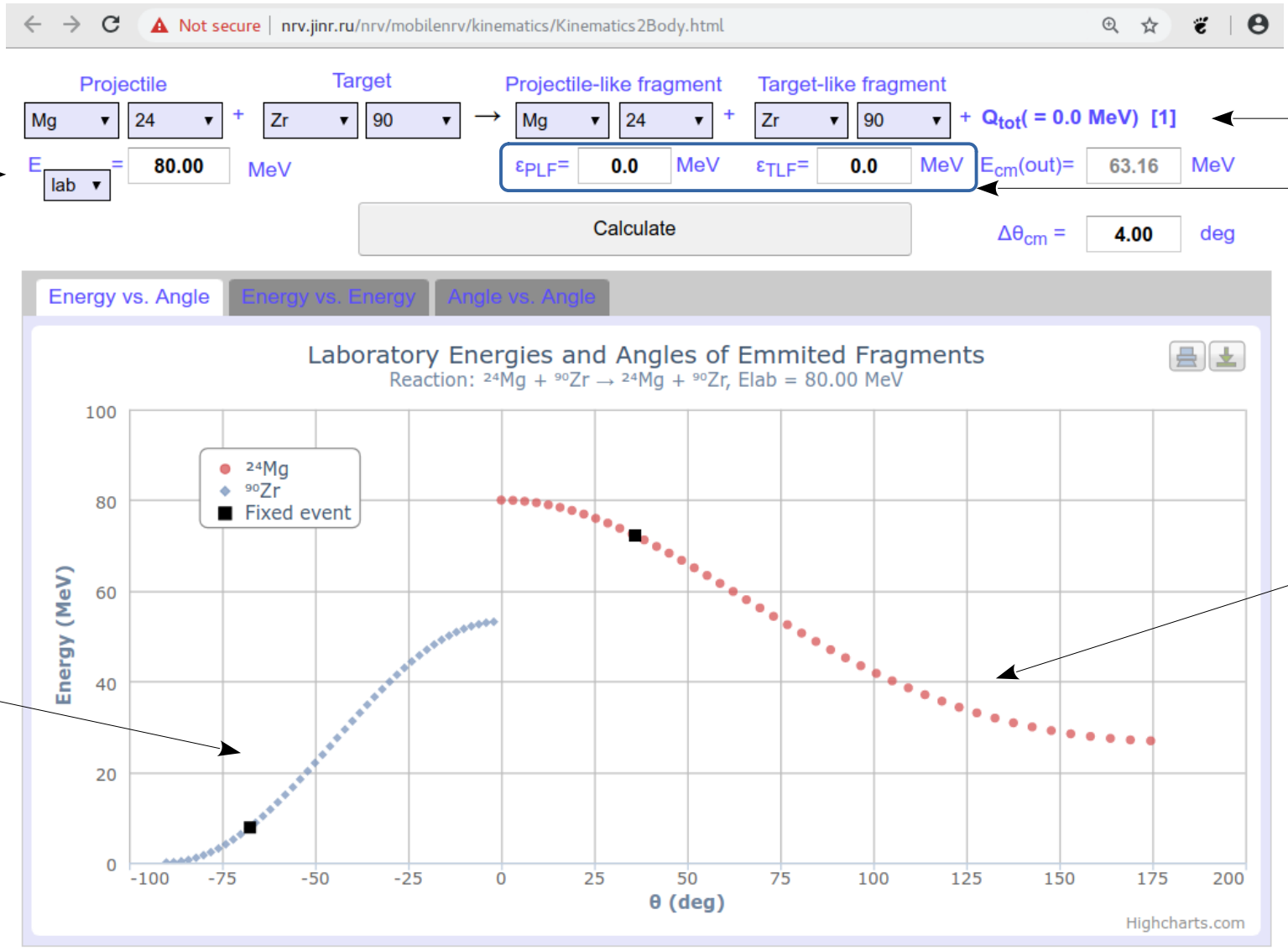
Kinetic energy of beam nucleus
Caution: press enter!

After typing nuclei and *E*, Q is shown automatically.

Excitation energies of PLF and TLF

PLF = Projectile-like fragment

TLF = Target-like fragment



PLF

TLF

Kinematically allowed values

	θ_{lab} (deg)	E_{lab} (MeV)
^{24}Mg	$0.0^o < \theta_{lab} < 180.00^o$	$26.81 < E_{lab} < 80.00$
^{90}Zr	$0.0^o < \theta_{lab} < 90.00^o$	$0.0 < E_{lab} < 53.19$

Fixed event parameters

E_{PLF}= 72.21 MeV θ_{PLF}= 35.99 deg

E_{TLF}= 7.79 MeV θ_{TLF}= -67.50 deg

Show/Hide θ_{cm}= 45.00 deg

Save data as ASCII file

Give some input values and you'll get output & black point on the plot.

In geom. model maximal (output) scattering angle before nuclei touch each other

Grazing collision[[4], p.7]: θ_{gr}(cm) ≈ 143.3 deg, θ_{gr}(lab) ≈ 131.9 deg, b_{gr} ≈ 1.7 fm, L_{gr} ≈ 13 ħ

Coulomb barrier[[4], p.326, 332]: V_C ≈ 62.2 MeV, R_C ≈ 10.3 fm

Package for fragment separators including utilities

- **LISE++** by O.B Tarasov and D. Bazin @ Michigan State University (MSU)

  Its main goal concerns **fragment separators** (transmission and yields of fragments produced / collected there). However, it offers several calculators, including the **Relativistic Kinematics Calculator**.

  WWW: **https://lise.nscl.msu.edu**

  Papers: [2008], [2016], [2023] .

  Installation:

  - Download: [**HERE**]
  - NPD training computer: first install on your account:

    ① `mkdir lise; cd lise`
    ② `cp ~kpiasecki/soft/lise++/ctnp/lise_setup.sh .`
    ③ `./lise_setup.sh`

    Then run using wine:

    ④ `nice wine ./LISE++.exe`

- **Instructions** on **Relativistic Kinematics calculator:** [**HERE**] and [**HERE**]

- **How to open Calculator**:

  - From the upper toolbar, choose:

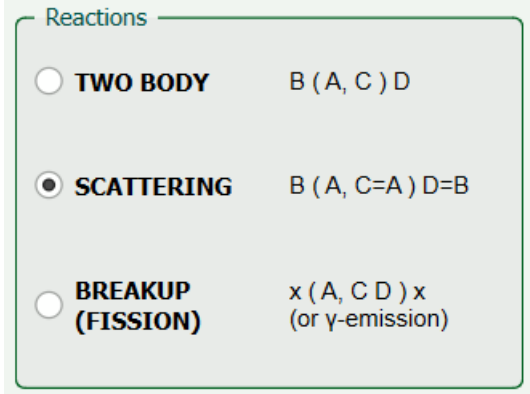


- Different **types of processes**:

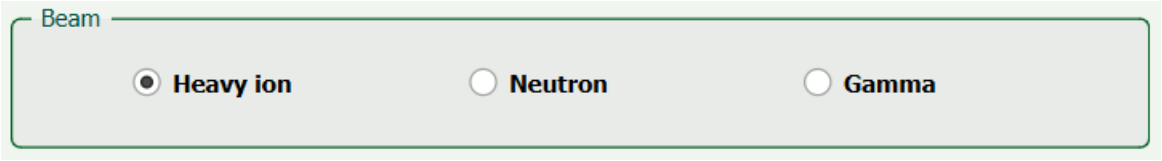
Scattering: A + B → A + B , nuclear convention: B (A, A) B

Two body: A + B → C + D , nuclear convention: B (A, C) D

Breakup (**fission**) , but also: decay.
Understood as „spontaneous" process, i.e. symbol x means none.
Either fission of nucleus (A → B C), or decay from excited state (A* → B C)



- Different **types of beam**:

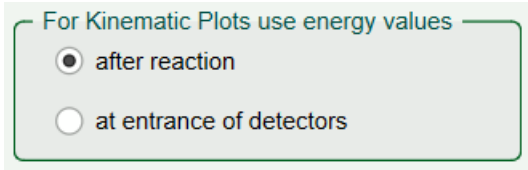


- Target has **thickness**
  ⇒ substrates and products **lose energy**.

Lise needs to be told,
where in the target the reaction occurs.

Also, if energy of products should be given
just after reaction, or after crossing the target.





Relativistic Kinematics Calculator

- **Example**

$^{24}Mg + {}^{90}Zr \rightarrow {}^{22}Mg + {}^{92}Zr$

T_{Beam} = 30A MeV

Target thickness: 1.5 μm

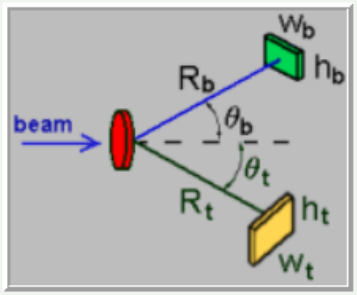
Reaction: mid-target

Click

Reactions

- **TWO BODY** B (A, C) D
- **SCATTERING** B (A, C=A) D=B
- **BREAKUP (FISSION)** x (A, C D) x (or γ-emission)

Fission (breakup) batch mode



For Kinematic Plots use energy values

- after reaction
- at entrance of detectors

Kinematics plots

Rutherford plot

2D fragment plot (Monte Carlo)

Beam

- **Heavy ion**
- **Neutron**
- **Gamma**

Participants

			ME [MeV]	Excitation Energy	E(CM) = 565.74 MeV	
A	Beam	^{24}Mg	-13.93	0	Beam energy	30 MeV/u
B	Target	^{90}Zr	-88.77	0	Intensity	1 kW
C	Fragment	^{22}Mg	-0.4	0	Target thickness	1.5 micron
D	Residual	^{92}Zr	-88.46	0.001	Q_value = -13.85 MeV	?
					Q_opt= +0.0 & Ex= -13.8 MeV	S

Reaction takes place at the target

- **Entrance**
- **Middle**
- **Exit**

Set-up

Search an angle in CM

- from 0 degrees and up
- from 180 degrees and down

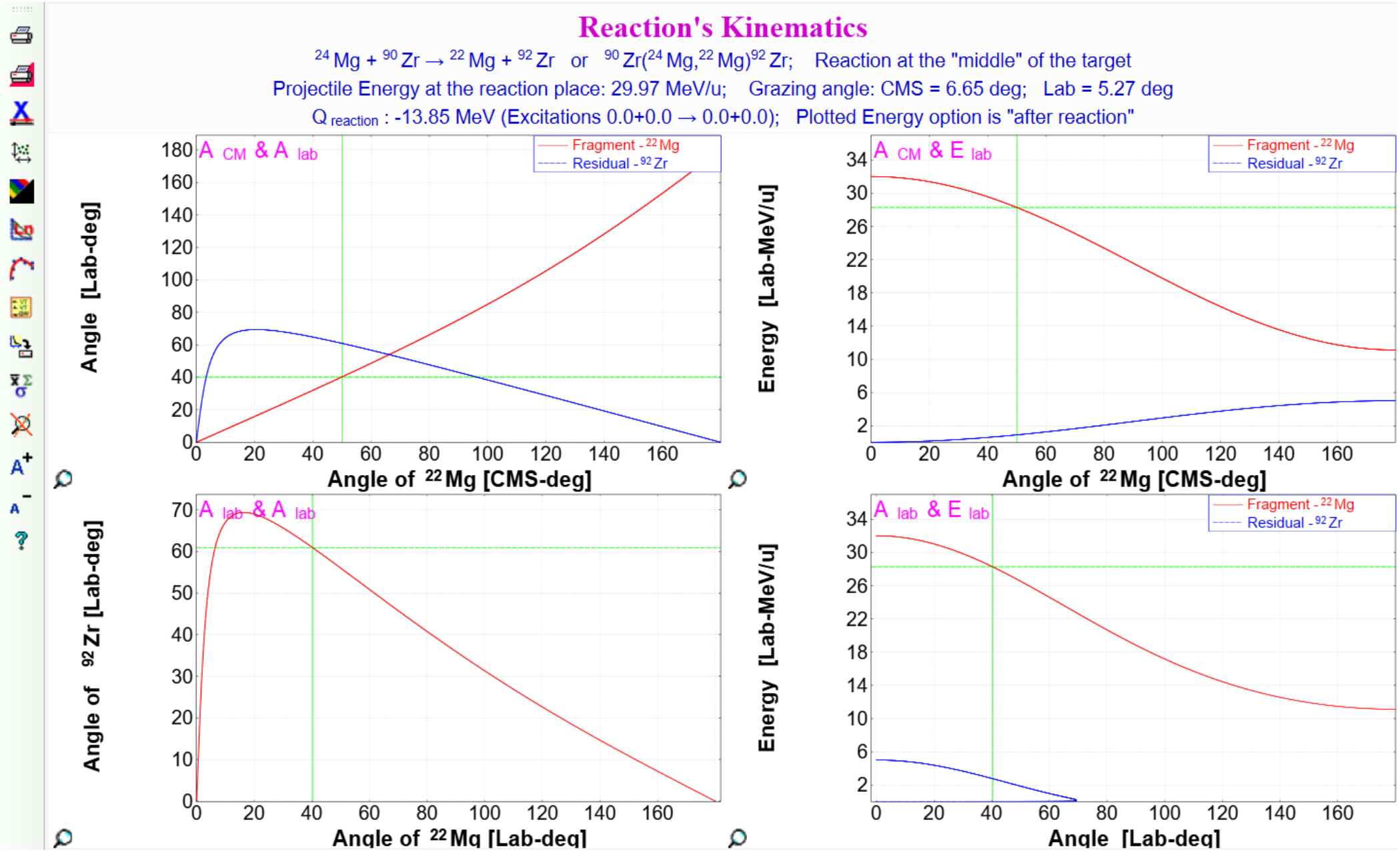
	fragment (C)		residual (D)
R =	10000	cm	10000
w =	1	cm	1
h =	2	cm	2

Selected participant

	LAB "C"	LAB "D"	CM "C"	CM "D"
Angle (deg) =	40.246	60.914	50	130
	fragment (C)	**residual (D)**	**fragment (C)**	**residual (D)**

Calculations

	LAB		CM		
Counting in monitor =	1.61e-01	2.42e-01			pps
Differential Cross Section =	142.547	214.689	100	100	mb/sr
Energy after reaction =	28.288	0.8988	20.2076	1.170	MeV/u**
Energy at det.entrance =	28.239	0.6427			MeV/u (** MeV for γ)



- Click to save data into text file.