

Audit Report

Exactly Solvable Schrödinger Operators Related to the Hypergeometric Equation

Jan Dereziński & Pedram Karimi — University of Warsaw

This report documents errors and quality issues found during a close reading of the LaTeX source file **solv-hyp-uncolored.tex** (238 KB, ~5,047 lines). Issues are grouped by severity. Line numbers refer to the source file.

A. Mathematical / Technical Errors

1. Wrong Wronskian formula [line 1830]

In the proof of the Hyperbolic Gegenbauer Hamiltonian theorem the Wronskian reads:

$$W(P_{-\alpha, \lambda}^h, P_{\alpha, \lambda}^h) = \sin(\alpha) / \pi$$

The argument of $\sin$ is missing the factor π . The correct formula, consistent with the analogous equation on line 1686 (spherical case) and line 2385 (hyperbolic first-kind case), is $\sin(\pi\alpha) / \pi$. Because this Wronskian is used to derive subsequent expressions in the proof, the error could propagate.

2. Repeated function in basis description [lines 2372–2374]

The proof of the Hyperbolic hypergeometric Hamiltonian of the first kind begins:

Considering $P_{-\alpha, \beta, \mu}^h(r)$, and $P_{-\alpha, \beta, \mu}^h(r)$ as a basis of solutions...

Both elements of the 'basis' are the same function ($P_{-\alpha}$ listed twice). The second entry should be $P_{\alpha, \beta, \mu}^h(r)$ — i.e., P_{α} , not $P_{-\alpha}$.

3. Possible spectrum error for $L_{\{NN\}}$ [line 3492]

The Dirichlet–Dirichlet Laplacian spectrum is written $\sigma(L_{\{DD\}}) = \{n^2 \mid n \in \mathbb{N}\}$ (using $\mathbb{N}$, i.e. $\{0, 1, 2, \dots\}$), while the Neumann–Neumann Laplacian has:

$$\sigma(L_{\{NN\}}) = \{n^2 \mid n \in \mathbb{N}\}$$

Since the paper uses both N and $\mathbb{N}$ as distinct symbols, N apparently denotes $\{1, 2, \dots\}$. But $-\partial^2_x$ on $]0, \pi[$ with Neumann conditions at both ends has $n = 0$ as an eigenvalue (the constant function), so the spectrum is $\{0, 1, 4, 9, \dots\}$. This requires $n \in \mathbb{N}$, matching $L_{\{DD\}}$.

4. Forward reference to `\eqref{hyper5}` [line 2331]

In the Hyperbolic subsection of Section 4, the text refers to 'the eigenvalue problem `\eqref{hyper5}`'. However, label `hyper5` is defined at line 2432, inside the DeSitterian subsection that follows. The correct label for the hyperbolic eigenvalue equation is `\eqref{hyper}`, defined at line 2263.

This is a genuine cross-reference error, not just a forward-reference style choice, because `hyper5` belongs to a different equation.

5. Inconsistent punctuation inside align for identities (pih1/pih2) [lines 574–579]

The two identities are formatted as:

```
K_{\tau,-1}^s = K_{\tau,1}^s , \tau \neq 0, \label{pih1} \quad K_{\kappa,-1}^h = K_{\kappa,1}^h . \kappa \neq 0.
\label{pih2}
```

pih1 closes with a comma; pih2 closes with a period inside the math environment and then appends $\kappa \neq 0$. with a further period after `\end{align}`. The condition $\kappa \neq 0$ is thereby set in math mode as if it were part of the equation, and the trailing period is duplicated. Both identities should be formatted uniformly.

B. LaTeX / Structural Errors

6. Duplicate `\label{riem}` [lines 2048 and 2055]

The label `riem` is assigned to two different equations in Section 4. LaTeX will emit a 'multiply-defined label' warning and any `\eqref{riem}` cross-references will silently resolve to the wrong equation.

7. Duplicate `\label{names}` [lines 1118 and 1121]

The table of Hamiltonian names has `\label{names}` specified twice: once inside the `\begin{table}[ht]` argument and again as a standalone command on the next line. One occurrence should be removed.

8. Duplicate 'we obtain' [lines 2380–2381]

A copy-paste artifact leaves two consecutive 'we obtain' phrases:

...arguing as in the proof of `\eqref{wrons}`, we obtain

we obtain

`\beq ...`

The second 'we obtain' should be deleted.

9. Thirty-six unresolved `\red ...` passages [41 occurrences total]

The file defines `\def\red{}` at line 121, so `\red` currently renders as nothing. Despite this, 36 substantive passages are still wrapped in `\red ...`, including complete mathematical remarks, table content, and large portions of proofs. These appear to mark text still under review. The paper cannot be considered a final draft until each is either committed (wrappers removed) or revised.

10. Confusing cross-reference in the historical note on Scarf [line 1088]

The text reads: 'The case of the Scarf Hamiltonian `\eqref{scarf-}` is a bit different. In the original paper, Scarf did not consider the Hamiltonian in `\eqref{scarf}`.' Two distinct labels (`scarf-` from the Introduction, `scarf` from Section 4) are used for essentially the same operator. The second reference is a long forward-reference and the sentence as written implies Scarf did not define the Hamiltonian named after him, which is confusing. The sentence should be rephrased.

C. Typos and Language Errors

Line	Error in source	Correction
304	solvable in terms Kummer's confluent equation	...in terms of Kummer's...

Line	Error in source	Correction
302	Each of the group is subdivided	Each group is subdivided (or: each of the groups)
355	In the later section these results will be discussed	In the following sections
377	hypergergeometric equation	hypergeometric equation
562	the following coincidences	the following coincidences
843	boundary conditon	boundary condition
889	discussed e.g. in. \cite{...}	discussed e.g. in \cite{...} (remove extra period)
1089	authors refered to it	authors referred to it
1454	one-dimesional	one-dimensional
2381	we obtain / we obtain (duplicate)	delete one occurrence
2695	The L^2 integrable condition	The L^2 integrability conditions

Inconsistency: The abstract (line 304) writes 'can be viewed as a sequel to [DL]' while the Introduction (line 346) writes 'can be viewed as *the* sequel to [DL]'. Choose one form and apply it consistently.

D. Presentational / Quality Issues

11. 'DeSitterian' vs. 'deSitterian' capitalization

The paper alternates between 'DeSitterian' (Figure 3 caption, TikZ node labels) and 'deSitterian' (throughout the main text). One form should be used consistently. The lower-case 'd' variant appears far more frequently and is used in the operators' formal definitions, so it is the natural choice.

12. Two commented-out TikZ figure environments

Alternative versions of Figure 1 (Gegenbauer equation on the w -plane) remain in the source as large comment blocks. These are harmless but add significant clutter (~60 lines each) and should be removed before submission.

13. `\eqref{hyper5}` forward reference creates reader confusion [see item 4]

Even setting aside the label error, the hyperbolic subsection lists four solution functions and directs the reader to 'eigenvalue problem `\eqref{hyper5}`' — which is the DeSitterian equation defined later. This will confuse readers following the text linearly. The eigenvalue problem should be stated or labeled locally.

Summary by Priority

Priority	Items
Must fix — mathematical correctness	A1, A2, A3, A4

Priority	Items
Must fix — LaTeX compilation	B6, B7, B8
Must fix before submission	B9 (36 unresolved <code>\red ...</code> passages)
Should fix — clarity / references	A5, B10
Minor — typos and grammar	C (12 items in table)
Polish	D11, D12, D13

Note: Citation correctness, bibliography completeness, and cross-references to external works (e.g. [DL], [DeGe]) were not verified, as those sources were not available. The mathematical correctness of the main theorems (spectra, Green function formulas, transmutation identities) was assessed structurally but not independently re-derived.