

Line-by-line check of *From Heun class equations to Painlevé equations*

Verification report and list of corrections

16 August 2026

Corrected source: `Fuch_jan_corrected.tex`. The full unified diff is reproduced in the Appendix.

1. What was verified by computation, not by eye

All checks below were done symbolically (sympy), exact arithmetic, residual required to be identically 0.

(a) **The isomonodromy setup (§4.1).** Re-derived (eq2), (eq.1), (eq.2) from (eq), (eq1). The two compatibility conditions (compa1.), (compa2.) come out exactly as printed. Correct.

(b) **The deformed equation (Prop. appa, eq. (h-bis-)).** At $z = \lambda$ the indices are 0, 2; the w^{-1} recursion forces $v_1 = \mu v_0$, and the w^0 coefficient then vanishes *identically in* μ , so v_2 is free and the singularity is non-logarithmic for every μ . Correct.

(c) **Theorem main0, condition (muu1).** Proved in full generality (abstract σ, τ, η): substituting (fir0) into (compa2.) gives **exactly** (muu1), with no remainder. So (compa2.) $\iff$ (muu1) modulo (fir0). Correct as printed.

(d) **Conditions (muu1), (muu2), (muu3) in all ten types.** For each of the eleven explicit parameter sets of §4.6–4.15, with c read off from the stated compatibility functions, all three conditions vanish identically in z . Correct as printed — including the unusual single power $(z - \lambda)^1$ in the denominator of (muu3), which is right.

(e) **The full chain, end to end.** For each of the eleven types I built p, q from (heudef), took a, b and H as printed, imposed $\dot{\lambda} = H_\mu$, $\dot{\mu} = -H_\lambda$, and evaluated (compa1.) and (compa2.) as rational functions of z :

type	(compa2.)	(compa1.)
$(\underline{111}; \underline{1}) \rightarrow \text{P VI}$	0	0
$(\underline{21}; \underline{1}) \rightarrow \text{P V}$	0	0
$(\frac{3}{2}\underline{1}; \underline{1}) \rightarrow \text{deg-P V}$	0	0
$(2; 2) \rightarrow \text{P III'}$	0	0
$(2; \frac{3}{2}) \rightarrow \text{deg-P III'}$	0	0
$(\frac{3}{2}; 2) \rightarrow \text{deg-P III'}$	0	0
$(\frac{3}{2}; \frac{3}{2}) \rightarrow \text{ddeg-P III'}$	0	0
$(\underline{1}; 3) \rightarrow \text{P IV}$	0	0
$(\underline{1}; \frac{5}{2}) \rightarrow \text{P 34}$	0	0

type	(compa2.)	(compa1.)
$(;4) \rightarrow \text{P II}$	0	0
$(;\frac{7}{2}) \rightarrow \text{P I}$	0	0

(f) **Hamiltonian** $\rightarrow$ **second-order equation**. Eliminated μ from each Hamiltonian and compared with the printed Painlevé equation and parameter identification. Ten of eleven matched exactly. The exception is P IV — see §2 below.

(g) **Conditions (moo1)–(moo3), (mii1)–(mii2) of Theorem main1**. Checked by hand in every case where they are used; all hold. The specializations $c = m(\lambda - s)\rho$ (Case A) and $c = m\sigma$ (Case B) reduce (muu1)–(muu3) to (moo1)–(moo3) and (mii1)–(mii2) correctly, including the non-obvious collapse $(\frac{\sigma'c}{\sigma})(\lambda) - c'(\lambda) = m\rho(\lambda)$.

(h) **Appendix, eq. (rea3)**. The stated form of (compa2.) after substituting (fir0) agrees exactly with my independent derivation.

(i) **Other spot checks that passed**: the homography formula (pde3); the three sandwiching formulas (tra1), (pde1), (pde2); the quadratic reduction (pde-sq), (pde-sq1) and the rank count $2m$ (both for $m \geq 1$ and $m = 0$); the recursion in the proof of Prop. **series1**; the generalized Fuchs relation (rho3) against Airy $(-\frac{1}{4} - \frac{1}{4} = \frac{3}{2} - 2)$; the three invariances of the Hamilton equations in the appendix, including the generating function $F_2 = \lambda\tilde{\mu}/t$ for $\tilde{\lambda} = t^{-1}\lambda$, $\tilde{\mu} = t\mu$; and the quadrature in §“Hamiltonian solvable in quadratures”.

(j) **Reference hygiene (live part only)**. 228 labels, no duplicates, no undefined `\ref`; 26 bibitems, all cited, nothing cited that is missing. Braces balance; environments nest correctly. Same after my edits.

2. The one mathematical error

§“From Heun (1;3) to Painlevé IV”, parameter identification below (P4).

Printed: $\alpha = -\kappa_0 + 2\theta_\infty - 1$. Correct value: $\alpha = -\kappa_0 + 2\theta_\infty + 1$.

From the paper’s own $H = 2\lambda\mu^2 - (\lambda^2 + 2t\lambda + 2\kappa_0)\mu + \theta_\infty\lambda$, writing $Q = \lambda^2 + 2t\lambda + 2\kappa_0$ and $P = 4\lambda\mu = \lambda' + Q$,

$$\lambda'' = \frac{\lambda'^2}{2\lambda} - \frac{Q^2}{2\lambda} + 2(\lambda + t)Q - 4\theta_\infty\lambda - 2\lambda = \frac{\lambda'^2}{2\lambda} + \frac{3}{2}\lambda^3 + 4t\lambda^2 + (2t^2 + 2\kappa_0 - 4\theta_\infty - 2)\lambda - \frac{2\kappa_0^2}{\lambda}.$$

Matching $2(t^2 - \alpha)\lambda$ gives $\alpha = -\kappa_0 + 2\theta_\infty + 1$. With the printed value the residual is -4λ . $\beta = -2\kappa_0^2$ is right. **Fixed in the corrected file.**

3. Corrections applied (23)

Every change below is wrapped in `\corr{...}` in the corrected source and therefore prints **in red**. The macro is defined in the preamble, immediately after `\newcommand{\red}{}`:

```
\newcommand{\corrcolor}{\color{red}}
%\newcommand{\corrcolor}{}
\DeclareRobustCommand{\corr}[1]{\{\corrcolor #1\}}
```

To get a clean, all-black submission version, comment out the first line and uncomment the second; nothing else changes. Correction 12 (the stray word *red*) is a pure deletion and so has nothing to mark.

Mathematical / structural:

1. **P IV**: α sign of the constant, as above.
2. **§3.1**: “at least one index equal 0 iff $c_1 = \dots = c_{n-1} = 0$ ” $\rightarrow c_1 = \dots = c_n = 0$. The sum in (req1-) runs to n , and (req2) drops all n of them.
3. **§3.1**: symbol of the M_n type, underbrace “ $n-1$ times” $\rightarrow$ “ n times”. There are n finite Fuchsian points plus ∞ .
4. **Proof of Thm. heuni, case** $\deg \sigma = 0$: “we can assume that $\tau(z) = z^3 + a_1 z + a_0$ ” $\rightarrow \eta(z) = b_3 z^3 + b_1 z + b_0$. $\tau = 0$ was established two lines earlier; the object being normalized is η , and the table’s ($\frac{7}{2}$) row confirms it.
5. **(thome1)**: $w_{0,k}, v_{0,k} \rightarrow w_{2,k}, v_{2,k}$. The two formal solutions are indexed $i = 1, 2$; there is no index 0.
6. **Proof of Prop. series1, item (4)**: the proof referred to (pwp), which is the condition of item (2). The condition actually used is $jp_{-l+1} + q_{-l} = 0$, i.e. item (4)’s $p_{-m}n + q_{-m-1} = 0$. That equation is now numbered (pwp2) and the reference corrected.
7. **Proof of Prop. series1, item (3)**: “If $m = 2$, there is an additional term $2v_0$ in (w2)” is not correct — at $n = -m + 1 = -1$ the second-derivative term is $(n+2)(n+1)v_{n+2} = 1 \cdot 0 \cdot v_1 = 0$. Replaced by a correct statement of what the v'' term contributes and why it is harmless.
8. **Definition of the grounded class** (Introduction, and a new remark in §3.4). Being grounded at all finite singularities is equivalent to η being a polynomial, which gives only $\deg \eta \leq 2n - 2 - \deg \sigma$; the condition $\deg \eta \leq n - 2$ of (req3-/) / (hground) is an extra normalization at ∞ and is automatic **only when** $\deg \sigma = n$. As printed, the text asserts an equivalence that fails whenever $\deg \sigma < n$. Concretely, the normal form ($\frac{1}{2}; \frac{5}{2}$) of Thm. **heuni** — $\sigma = z, \tau = a_0, \eta = b_2 z^2 + b_1 z + b_0$ — is grounded at $z = 0$ (indices 0 and $1 - a_0$) but has $\deg \eta = 2 > 1$, so it is not in the grounded Heun class as defined. I corrected the Introduction sentence and added a short remark after (hground); please check the wording is what you intend, since this is a definitional choice, not just a typo.
9. **Prop. appa proof**: the displayed powers $(z - \lambda)^0, (z - \lambda)^1, (z - \lambda)^{n-1}$ are correct only after multiplying (h-bis-) by $(z - \lambda)$. Said so explicitly.
10. **deg-P V**: the “Set” block had $\eta = -\frac{t}{z-1} + \frac{(\kappa_0^2 - \kappa_\infty^2)z}{4} - c$ while the displayed equation and (P5dh) use $(z - 1)$ in place of z . Harmless (absorbed in c), but made consistent.
11. **§2.4**: “For $r(z) = \kappa \log(z)$ ” $\rightarrow \kappa \log(z - z_0)$, matching (tra1).

Typography / LaTeX:

12. **L566**: stray word **red** in running text (“ M_n class equations. *red* It is easy to see”) — this prints. Removed.
13. **(P3p1H)**: $\backslash\mathrm{big}(1-\backslash\mathrm{chi}_0)\backslash\mathrm{lambda}+t\backslash\mathrm{big}) \rightarrow \backslash\mathrm{big}((1-\backslash\mathrm{chi}_0)\backslash\mathrm{lambda}+t\backslash\mathrm{big})$. As printed it renders as a stray closing parenthesis and misreads the coefficient. 14–15. **(eq.1), (eq.2)**: $\backslash\mathrm{big}(\mathrm{closed\ by\ a\ plain\ })$.
14. **Appendix (rea3)**: $\backslash\mathrm{big}(\mathrm{cp}_0)'(z) \rightarrow \backslash\mathrm{big}(\mathrm{cp}_0\backslash\mathrm{big})'(z)$. 17–22. Six occurrences of **Painleve** / **Painlev'e** in *printed* text (two subsection titles, the $\diamond$ line of §5.3, and three body sentences) $\rightarrow$ **Painlev\’e**. The $\backslash\mathrm{label}\{\}$ keys **Painleve** IV-34 and **section: Painleve** were deliberately left untouched, since they are referenced elsewhere.
15. “isomodromic” $\rightarrow$ “isomonodromic”; “Hence it works in under generic conditions” $\rightarrow$ “works under”; “it does not works” $\rightarrow$ “does not work”.

4. Flagged but not changed — your call

- **1056 lines after `\end{document}` (line 5109).** They contain a complete earlier paper on *derivative Heun class equations*: its own `\begin{abstract}`, `\section{Introduction}`, five subsections on derivative Heun (111;1), (12;1), (1;3), (2;2), (;4), acknowledgements, a second bibliography, a second `\appendix` with “Compatibility conditions”, a second `\end{document}`, and an orphan `\subsection{...}` stub. Inert in the PDF. But the live Introduction still discusses derivative Heun class equations and Kimura’s gauge result while the live body never treats them — either cut that passage or reinstate the section.
 - `{\red ... }` **block** in the Introduction (the A1/Ap/Aq/Bp/Bq discussion). `\red` is `\newcommand{\red}{}`, so it prints normally; it presumably marks text you were still revising.
 - `\ArticleDates{Received ???, in final form ???; Published online ???}` — SIGMA template placeholder.
 - **Two author lists.** The preamble `\author{}` orders Dereziński, Latosiński, Ishkhanyan; the `\Author{}` actually used by the class orders Dereziński, Ishkhanyan, Latosiński. Also a commented-out Filipuk affiliation.
 - **Notation slip:** the deg-P V subsection writes “Heun ($\frac{3}{2}1;1$) equation” where the rest of the paper writes ($\frac{3}{2}\underline{1};\underline{1}$).
 - (;4) **normal form:** the proof says “by scaling we can make $a_2 = 1$ ” while the table records $a_2 = c$ arbitrary nonzero. Consistent, but the proof is more restrictive than the statement.
 - **The claim of unification.** Theorem `main0`’s hypotheses are three functional identities in (z, λ) constraining t and c ; Theorem `main` still splits into A1/Ap/Aq/Bp/Bq. The real reduction — a genuine one — is that the case analysis collapses to the choice of the degree- ≤ 2 polynomial c , and c is dictated by where σ degenerates. “Automatize these derivations” and “a unified algorithm” oversell it and will draw a referee’s fire; I would state the weaker, defensible claim.
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Appendix. Unified diff of the 23 corrections

Against the original `Fuch_jan.tex`. Red lines are the original, green lines the corrected file. Lines longer than 90 characters have been wrapped, with the leading `-/+` repeated on the continuation; use `corrections.diff` for the applicable patch. The `\corr{...}` wrappers are presentational only – see the note at the head of Section 3.

```
--- orig.tex      2026-08-16 10:37:44.794177704 +0000
+++ work.tex      2026-08-19 15:27:52.110004545 +0000
@@ -45,6 +45,15 @@
%\refcheck

\newcommand{\red}{}
+
+%% Marking of the corrections -----
+%% Everything changed or added with respect to the original Fuch_jan.tex
+%% is wrapped in \corr{...}. To obtain a clean, all-black version,
+%% comment out the first line below and uncomment the second one.
+\newcommand{\corrcolor}{\color{red}}
+%\newcommand{\corrcolor}{}

```

```

+\DeclareRobustCommand{\corr}[1]{\corrcolor #1}
+%%%%% -----
\newcommand{\blue}{\color{blue}}

@@ -563,7 +572,7 @@
Following
\cite{Sl_Lay}, the class of equations with  $n$  singularities in
 $\mathbb{C}$  and a singularity at infinity, and their confluent cases are
-called {\em  $M_n$  class} equations. red It is easy to see that these equations can be
+called {\em  $M_n$  class} equations. It is easy to see that these equations can be
written as
\begin{equation}
\big(\sigma(z)\partial_z^2+\tau(z)\partial_z+\eta(z)\big)u(z)=0,
@@ -575,7 +584,11 @@
which one of indices of all finite singularities is zero---hence the
name ``grounded''. (In \cite{Sl_Lay} such equations are called {\em
canonical}. In our opinion, the word canonical is overused, hence
-not appropriate for this meaning.) They can be written as \eqref{pde.}
+not appropriate for this meaning.) \corr{Being grounded at all finite singularities means
+precisely that  $\eta$  is a
+polynomial, which by itself gives only  $\deg \eta \leq 2n-2-\deg \sigma$ . We impose in
+addition the normalization  $\deg \eta \leq n-2$ , which makes the class invariant under
+interchanging a finite singularity with  $\infty$ ; note that it is automatic only when
+ $\deg \sigma=n$ .} Thus grounded  $M_n$  class equations can be written as \eqref{pde.}
with the same conditions on  $\sigma, \tau$ , and with  $\eta$  a
polynomial of degree  $\leq n-2$ .

@@ -743,7 +756,7 @@
is unified, however it is not very transparent.
Theorem \ref{main1}, which can be viewed as the first part of the main
result of this paper, is more satisfactory. The method of
-isomodromic deformation is here divided into two slightly different
+\corr{isomonodromic} deformation is here divided into two slightly different
ansatzes, called Case A and B. The main
condition on
 $\sigma, \tau, \eta, t$  for the applicability of Ansatz A is the existence of a zero of
@@ -802,9 +815,9 @@

Note that Subcase A1 works in the
standard Heun
-type  $(\infty, \infty, \infty, \infty)$ , leading to Painlevé VI. Hence it works in
+type  $(\infty, \infty, \infty, \infty)$ , leading to Painlevé VI. Hence it \corr{works}
under generic conditions.
-However, it does not works for some confluent types. For instance, for most degenerate
+However, it does not \corr{work} for some confluent types. For instance, for most
+degenerate
types one needs to use either Subcase Aq or Subcase Bq.

```

Altogether, Theorem \ref{main} works for certain normal forms of all types of Heun class equations. This

@@ -1056,7 +1069,7 @@

```
\noindent
{\bf Sandwiching with powers.}
-For  $r(z)=\kappa\log(z)$ 
+For  $r(z)=\kappa\log(z\corr{-z_0})$ 
\begin{align} & (z-z_0)^{-\kappa} \\
& \left( \partial_z^2 + p(z)\partial_z + q(z) \right) \\
& (z-z_0)^{\kappa} \label{tra1} \notag \end{align}
```

@@ -1307,7 +1320,7 @@

```
    If  $m \geq 2$  and  $l = m+1$ , then there
    is no power series solution unless
    for some  $n=1,2,\dots$ 
-     $[p_{-m}n + q_{-m-1}] = 0.$ 
+    \begin{equation} \corr{p_{-m}n + q_{-m-1} = 0.} \label{pwp2} \end{equation}
    \item
    If  $m \geq 1$  and  $l \geq m+2$ , then
    there are no power series solutions.
```

@@ -1357,7 +1370,7 @@

```
     $0 = (jp_{-l+1} + q_{-l})v_j + \cdots,$ 
\end{align}
    where dots denote terms depending on  $v_{j-1}, \dots, v_0$ .
- If (\ref{pwp}) has no solutions, then we can solve the recurrence obtaining
+ If \corr{(\ref{pwp2})} has no solution  $n=1,2,\dots$ , then we can solve the recurrence
+ obtaining
     $v_0 = v_1 = \dots$ .
```

Let us prove (3). We have the recursion relations

@@ -1368,7 +1381,7 @@

```
     $0 = jp_{-m}v_j + \cdots,$ 
\end{align}
    where  $\cdots$  involve  $v_0, \dots, v_{j-1}$ .
- If  $m=2$ , there is an additional term  $2v_0$  in (\ref{w2}).
+ \corr{If  $m=2$ , the second derivative contributes an additional term}
+  $(j+1-m)(j-m)v_{j+1-m}$  to the  $j$ -th relation; since  $m \geq 2$  it involves only  $v_k$  with
+  $k \leq j-1$ .}
    Note that  $p_{-m} \neq 0$ .
    Hence, for any  $v_0$  we can solve the recurrence obtaining  $v_1, v_2, \dots$ .
```

@@ -1433,7 +1446,7 @@

```
\end{align}
    Here  $\mathop{\sum}$  denotes the sum where the index  $k$  within its range runs over
    both integers and half-integers. We have
-     $w_{0,k} = (-1)^{2k}w_{1,k}, \quad v_{0,k} = (-1)^{2k}v_{1,k}.$ 
+     $\corr{w_{2,k}} = (-1)^{2k}w_{1,k}, \quad \corr{v_{2,k}} = (-1)^{2k}v_{1,k}.$ 
    \label{thome1} \eeq
```

```

\end{enumerate}
\label{thome} \end{proposition}
@@ -1623,7 +1636,7 @@
\proof
We look for a solution analytic around  $\lambda$ :

$$v(z) = \sum_{n=0}^{\infty} v_n(z-\lambda)^n.$$

-We obtain
+ \corr{Multiplying (\ref{h-bis-}) by  $(z-\lambda)$  and inserting the series} we obtain
\begin{align*}
& \big(-\sigma(\lambda)v_1 + \mu\sigma(\lambda)v_0\big)(z-\lambda)^0 \\
& + \Big(\sigma(\lambda)^2v_2 + \big(\tau(\lambda) - \sigma'(\lambda)\big)v_1 - \sigma(\lambda)^2v_2 \\
& - \big(\mu\tau(\lambda) - \mu\sigma'(\lambda) + \mu^2\sigma(\lambda)\big)v_0 + \mu\sigma(\lambda)v_1\Big)(z-\lambda)^1 \\
& + \sum_{n=3}^{\infty} \Big(\sigma(\lambda)n(n-2)v_n + \dots\Big)(z-\lambda)^{n-1}; = 0
\end{align*}
@@ -1684,10 +1697,10 @@
where  $z_1, \dots, z_n$  are distinct points in  $\mathbb{C}$ .
The family of equations (\ref{req1-}) will be called {\em the  $M_n$  type}.
The corresponding symbol is

$$-(\underbrace{\text{ times }}_{\dots}); \underbrace{\text{ times }}_{\dots}).$$


$$+(\underbrace{\text{ times }}_{\dots}); \underbrace{\text{ times }}_{\dots}).$$


Each finite singularity has at least one index equal 0 if and only if

$$c_1 = \dots = c_{n-1} = 0.$$


$$c_1 = \dots = \corr{c_n} = 0.$$

Thus such equations are given by operators
\begin{equation}
\partial_z^2 + \sum_{j=1}^n \frac{a_j}{z-z_j} \partial_z
\end{equation}
@@ -1913,6 +1926,15 @@
If in addition  $\sigma$  has 3 distinct roots, then
(\ref{req3aa}) is a (grounded) Heun type operator.

+ \begin{remark} \corrcolor Being grounded at all finite singularities amounts to  $\sigma$ 
+ dividing
+  $\xi$ , that is, to  $\eta$  being a polynomial; this alone gives only
+  $\deg \eta \leq 4 - \deg \sigma$ . The condition  $\deg \eta \leq 1$  in \eqref{hground} is an
+ additional normalization at  $\infty$ , and it is automatic only if  $\deg \sigma = 3$ .
+ Consequently, some normal forms of Theorem \ref{heuni}---for instance
+  $(\underline{1}; \frac{52}{2})$ , where  $\sigma(z) = z$  and  $\deg \eta = 2$ ---are grounded at all
+ their finite singularities without belonging to the grounded Heun class.
+ \end{remark}
+

Clearly, the Heun class and the grounded Heun class are preserved
@@ -2257,7 +2279,7 @@
 $e^{-\kappa z} \cdot e^{\kappa z}$  with
 $\kappa = -\frac{a_0}{2}$ 
we kill  $a_0$ . Thus  $\tau = 0$ . If  $b_3 \neq 0$ , then

```

-after a transformation $z \mapsto az+b$ we can assume that $\tau(z)=z^3+a_1z+a_0$. We obtain the normal form of $(\frac{72}{72})$.
+after a transformation $z \mapsto az+b$ we can assume that $\eta(z)=b_3z^3+b_1z+b_0$. We obtain the normal form of $(\frac{72}{72})$.
If $b_3=0$, we obtain an operator that can degenerate to the Hermite, Airy, 1d Helmholtz or 1d Laplace type, all of the Riemann class.

@@ -2320,9 +2342,9 @@

```
\begin{align}\notag
\dot v'=&
\big(-\dot p-pa'+ap^2-pb-qa\big)v'\\
- & \big(paq-pb'-\dot q-qv, \quad \label{eq.2}\notag
+ & \big(paq-pb'-\dot q-qv\corr{\big}\big)v, \quad \label{eq.2}\notag
\dot v'=&
- & \big(-2qa'-aq'+b'+apq-bq\big)v' \quad \big(a''-2pa'-ap'+2b'-aq+ap^2-bp\big)v'. \label{eq.1}
+ & \big(-2qa'-aq'+b'+apq-bq\big)v' \\
+ & \big(a''-2pa'-ap'+2b'-aq+ap^2-bp\corr{\big}\big)v'. \label{eq.1}
\end{align}
Equating (\ref{eq.1}) and (\ref{eq.2}) we obtain
\begin{align}
```

@@ -3058,7 +3080,7 @@

```
Set
\begin{align}\sigma(z)&=(z-1)^2z, \tau(z)=(z-1)z+(1-\kappa_0)(z-1)^2, \\
- & \eta(z)=-\frac{t}{(z-1)}+\frac{\big(\kappa_0^2-\kappa_\infty^2\big)z^4}{4}-c. \\
+ & \eta(z)=-\frac{t}{(z-1)}+\frac{\big(\kappa_0^2-\kappa_\infty^2\big)\corr{(z-1)}}{4}-c. \\
\end{align}
```

Heun $(\frac{321}{1})$ equation
(with the singularity of rank 2 put at 1):

@@ -3175,7 +3197,7 @@

```
Type Ap compatibility functions
[a(z):=\frac{\lambda z}{t(z-\lambda)}, \quad b(z)=-\frac{\lambda^2\mu}{t(z-\lambda)}].
Painlev'e  $(2;\frac{32}{32})$  Hamiltonian:
-\begin{align}tH=&\lambda^2\mu^2+\big(1-\chi_0\big)\lambda+ t\big)\mu+\frac{\lambda}{2}. \\
+\begin{align}tH=&\lambda^2\mu^2+\corr{\big((1-\chi_0)\lambda+ t\big)}\mu+\frac{\lambda}{2}. \\
& \label{P3p1H} \end{align}
Painlev'e  $(2;\frac{32}{32})$  equation:
\begin{equation}\label{P3p1}
```

@@ -3298,7 +3320,7 @@

```
\frac{d^2\lambda}{dt^2}=\frac{1}{2\lambda} \left(\frac{d\lambda}{dt}t\right)^2+\frac{3}{2}\lambda^3+4t \lambda^2+2(t^2-\alpha)\lambda+\frac{\beta}{\lambda},
\end{eqnarray}
where
- $\alpha=-\kappa_0+2\theta_\infty-1$ ,  $\beta=-2\kappa_0^2$ .
+ $\alpha=-\kappa_0+2\theta_\infty\corr{+1}$ ,  $\beta=-2\kappa_0^2$ .
In the standard terminology (\ref{P4}) is called
the Painlev'e IV equation.
```

@@ -3496,7 +3518,7 @@

\label{Painlev\'e V}
%Heun $(\underline{}\underline{})$

-As noted in \cite{Ohyama}, the usual form of the Painlev\'e V equation, depending on 4 parameters, should be treated not as a single type, but as a supertype. It is invariant with respect to a scaling transformation.

+As noted in \cite{Ohyama}, the usual form of the \corr{Painlev\'e} V equation, depending on 4 parameters, should be treated not as a single type, but as a supertype. It is invariant with respect to a scaling transformation.

It includes two nontrivial types: nondegenerate V depending on 3 parameters and degenerate V depending on 2 parameters. There exists also a trivial type, solvable in quadrature.

In this subsection we discuss the supertype Painlev\'e V in detail. Note that we prefer to denote it by $(\underline{}\underline{})$, since it corresponds to

@@ -3624,7 +3646,7 @@

We prefer to denote it by the symbol $(\underline{}\underline{})$, because it corresponds to the supertype $(\underline{}\underline{})$ of the Heun class.

\noindent

-\$\diamond\$ Painleve III' or $(\underline{}\underline{})$ equation and Hamiltonian:
+ $\diamond$ \corr{Painlev\'e} III' or $(\underline{}\underline{})$ equation and
+Hamiltonian:

\begin{align}
\frac{d^2\lambda}{dt^2}&=\frac{1}{\lambda}\left(\frac{d\lambda}{dt}\right)^2-\frac{1}{t}\frac{d\lambda}{dt}+
\frac{\alpha\lambda^2+\gamma\lambda^3}{4t^2}+\frac{\beta}{4t}+\frac{\delta}{4\lambda},\label{painleve3}\\&\end{align}

@@ -3784,7 +3806,7 @@

The Painlev\'e III' equation is obtained from (\ref{P3}) by $\tilde{\lambda}=t\lambda$, $\tilde{t}=t^2$.
\end{remark}

-\subsection{Painleve IV-34 or $(\underline{}\underline{}\underline{})$ }

+\subsection{\corr{Painlev\'e} IV-34 or $(\underline{}\underline{}\underline{})$ }

\label{Painleve IV-34}

@@ -3891,7 +3913,7 @@

Then we change t into $-t$ and multiply the Hamiltonian by -1 , obtaining the Painlev\'e (4) Hamiltonian (\ref{H2-}) with $\kappa_0=\alpha+\frac{1}{2}$.
\end{remark}

-\subsection{Painleve I-II or $(\underline{}\underline{}\underline{}\underline{})$ }

+\subsection{\corr{Painlev\'e} I-II or $(\underline{}\underline{}\underline{}\underline{})$ }

Usually Painlev\'e I and II equations are treated separately.

However, it has been noted already by Painlev\'e and elaborated in \cite{Ohyama} that it

is natural to join them
in a single supertype. \cite{Ohyama} denotes it $\underline{\{;\};2}$, we denote it I-II,
or preferably $\underline{4}$, since it corresponds to the supertype $\underline{4}$

@@ -3947,12 +3969,12 @@

% & $H_{\{1,\beta\}}(t+6\beta^2,\lambda-\beta,\mu-2\beta\lambda+4\beta^2)\backslash$
 $H=\frac{\tilde{\mu}^2}{2}-\text{Big}(\tilde{\lambda}^2+\frac{\tilde{t}^2}{2}\text{Big})\mu-\text{Big}(2\beta^3+\frac{1}{2}\text{Big})\tilde{\lambda}+\frac{1}{2}\beta-\beta^2\tilde{t}.$
\end{align}

Thus up to free terms we obtain the Painlevé (4) Hamiltonian with
- $\alpha=2\beta^3$, and hence also the Painlevé (4) equation.
+ $\alpha=2\beta^3$, and hence also the \corr{Painlevé} (4) equation.

\item
Let $\eta=0$, $\beta\neq 0$. By
scaling we can set $\beta=1$. The Hamiltonian becomes
 $[H=\frac{\mu^2}{2}-2\lambda^3-t\lambda.]$
- Thus we obtain the Painlevé $(\frac{7}{2})$ Hamiltonian, and hence also the Painlevé
- $(\frac{7}{2})$ equation.
+ Thus we obtain the Painlevé $(\frac{7}{2})$ Hamiltonian, and hence also the
+ \corr{Painlevé} $(\frac{7}{2})$ equation.

\item
Let $\eta=0$, $\beta=0$. The Hamiltonian becomes
 $H=\frac{1}{2}\mu^2,$

@@ -4076,7 +4098,7 @@

-c'(z)+c'(\lambda)+c''(z)(z-\lambda)}{(z-\lambda)^2}

%\label{mu1a.}

\\label{rea3}

-& +\frac{(cp_0(z)-(cp_0(\lambda)-(z-\lambda)\text{big}(cp_0)'(z))}{

+& +\frac{(cp_0(z)-(cp_0(\lambda)-(z-\lambda)\text{big}(cp_0\corr{\text{big}})'(z))}{

{(z-\lambda)^2} + \dot{p}_0(z).

%\label{mu1.}

\end{align}