

Mathematical Audit

Dereziński, Group-theoretical origin of symmetries of hypergeometric
class equations and functions

Independent equation-and-theorem review

30 July 2026

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Scope and verdict

I audited the supplied 137-page PDF, including all 718 distinct numbered equation labels and every theorem, proposition, lemma, and corollary. I also checked the unnumbered transmutation and factorization tables, because several of the paper's most consequential mistakes occur there.

The central group-theoretic program is mathematically sound, and most coordinate reductions, differential operators, standard-function definitions, recurrence relations, and connection formulas are consistent. The paper is **not reliable verbatim**, however. It contains:

- several false operator identities;
- an incorrect Casimir and incorrect consequences derived from it;
- a theorem with an insufficient (and algebraically wrong) endpoint condition;
- a proposition whose own proof requires a hypothesis absent from its statement;
- a noncommutative subalgebra called “maximal commutative”;
- an incorrect semidirect-product description of the Heisenberg algebra;
- repeated missing branch, convergence, and differentiation hypotheses.

The most serious problems are concentrated on pp. 55-60, 78-81, 89, 103, 122, and 132.

Method

The review used four checks:

1. Every numbered label was indexed by section.
2. First- and second-order differential-operator identities were independently composed symbolically, including parameter shifts.
3. Casimir and heat-operator identities were checked directly in the displayed differential representations.
4. Every suspected error was verified visually in the rendered PDF, so none of the findings below is caused by text extraction.

For integral theorems, I applied the differential operator to the integrand and checked the claimed total derivative. Standard hypergeometric and confluent connection formulas were also checked at generic complex parameter values.

I. Definite mathematical errors

1. Invalid definitions of pullback and Lie derivative (p. 30)

The displayed definition

$$(\alpha^*g)(Y, Y) := g(\alpha(Y), \alpha(Y))$$

is not a valid coordinate-free pullback formula as written: it needs the differential

$d\alpha$ and the base-point composition. More seriously,

$$(\mathcal{L}_X g)(Y, Y) := g([X, Y], Y) + g(Y, [X, Y])$$

is not the Lie derivative of a metric. With the standard convention,

$$(\mathcal{L}_X g)(Y, Y) = X(g(Y, Y)) - 2g([X, Y], Y).$$

The printed expression has the wrong sign and omits the $X(g(Y, Y))$ term. It is not tensorial in Y . Later flat-coordinate computations do not appear to rely literally on this bad definition, but the foundational statement is false.

2. Equations (4.7) and (4.8) have the map in the wrong direction (p. 34)

The paper writes a map from a projective class

$$\mathcal{Y} \ni \mathbb{R}^\times(\rho, \omega) \mapsto (\rho, \omega) \in S^q \times S^p$$

and calls it a double covering. A projective class has two normalized representatives, so this is not a single-valued map. The well-defined double covering is the reverse map:

$$S^q \times S^p \longrightarrow \mathcal{Y}, \quad (\rho, \omega) \mapsto \mathbb{R}^\times(\rho, \omega).$$

The same reversal is needed in (4.8). Consequently, the following claim that (4.7) is a section is also false globally; it is a two-valued lift, or a genuine section only after passing to the double cover.

Related terminology: $\mathcal{V} \rightarrow \mathcal{Y}$ is naturally a principal $\mathbb{R}^\times$ -bundle (equivalently, the zero-removed tautological line bundle), not literally a line bundle with zero fibers included.

3. False domain-invariance claim before (4.24) (p. 41)

For a generic open $\mathcal{V}_0 \subset \mathcal{V}$, an arbitrary $\alpha \in O(n+2)$ does not preserve $\Lambda_+^\eta(\mathcal{V}_0)$ unless $\alpha(\mathcal{V}_0) = \mathcal{V}_0$. The later intersection domains partially repair this, but the blanket preservation statement is false.

4. Two wrong hypergeometric transmutation identities (pp. 55-56)

Let $D = \partial_w$.

- In the line shifting $(\beta, \mu) \mapsto (\beta - 1, \mu - 1)$, the intertwiner contains

$$w(w-1)D + \frac{1}{2}(w-1)(\alpha + \beta - \mu + 1) - \beta.$$

The last term must be $+\beta$, on both sides of the identity.

- In the line shifting $(\alpha, \mu) \mapsto (\alpha - 1, \mu - 1)$, the intertwiner contains

$$w(w-1)D + \frac{1}{2}w(\alpha + \beta - \mu + 1) + \alpha.$$

The last term must be $-\alpha$, on both sides.

The recurrence table later in the paper uses the corrected signs, which confirms that these are printing errors rather than a convention change.

5. All four $(w-1)\mathcal{F}_{\alpha,\beta,\mu}$ factorizations on p. 60 are wrong

For each of the four displayed lines, let P denote the printed product of first-order operators and let C denote the printed scalar product divided by 4. The paper prints the pattern

$$(w-1)\mathcal{F}_{\alpha,\beta,\mu} = P - C.$$

The correct identity is

$$(1-w)\mathcal{F}_{\alpha,\beta,\mu} = P + C.$$

Thus both the sign of the left side and the sign of the scalar remainder must change in all four lines. Direct composition shows that the printed second-order coefficient already has the opposite sign from the left side.

6. Proposition 5.3 uses the wrong endpoint power (pp. 64-65)

Write

$$A(\tau) = (z_1 - \tau^{-1}z_{-2})^{\alpha+\nu}(z_2 + \tau^{-1}z_{-1})^{\beta+\nu}.$$

Condition (5.29) is printed as

$$A(\tau)\tau^\nu|_{\tau(0)}^{\tau(1)} = 0.$$

For (5.30c), direct differentiation gives

$$B_{12}(A(\tau)\tau^{\nu-1}) - (\nu+1)A(\tau)\tau^\nu = -\partial_\tau(A(\tau)\tau^{\nu+1}).$$

Therefore the required endpoint condition is

$$A(\tau)\tau^{\nu+1}|_{\tau(0)}^{\tau(1)} = 0,$$

not (5.29) as printed. The proof also incorrectly says that both (5.30c) and (5.30d) use (5.26). In fact, (5.30c) uses the corrected (5.29), while (5.30d) uses (5.26).

The recurrence relation itself is correct once the endpoint condition is repaired.

7. Three wrong Gegenbauer transmutation lines (p. 78)

In the last four transmutation identities:

- the line printed with target $\mathcal{S}_{\alpha-1,\lambda+1}$ has the operator for $\mathcal{S}_{\alpha-1,\lambda-1}$;
- the line printed with target $\mathcal{S}_{\alpha-1,\lambda-1}$ has the operator for $\mathcal{S}_{\alpha-1,\lambda+1}$;

- in the line shifting to $\mathcal{S}_{\alpha+1, \lambda+1}$, the repeated intertwiner

$$wD + \alpha - \lambda + \frac{1}{2}$$

must be

$$\boxed{wD + \alpha + \lambda + \frac{1}{2}}.$$

Thus the two λ -targets are interchanged, and a third line has the wrong sign of λ .

8. Two wrong scalar signs in the Gegenbauer factorizations (p. 81)

In both factorizations beginning with

$$(1 - w^2)\mathcal{S}_{\alpha, \lambda},$$

the displayed product of first-order operators is correct, but the scalar remainder is not. Replace the final $+C$ by $-C$ in both lines.

The preceding factorizations of $\mathcal{S}_{\alpha, \lambda}$ and the following four factorizations of $w^2\mathcal{S}_{\alpha, \lambda}$ check correctly.

9. The Heisenberg algebra is not the semidirect product in (7.6)

Equation (7.6) writes

$$\text{heis}(2(n-2)) = \mathbb{R} \rtimes (\mathbb{R}^2 \otimes \mathbb{R}^{n-2}).$$

The Heisenberg algebra is a **non-split central extension** determined by the symplectic form:

$$0 \longrightarrow \mathbb{R} \longrightarrow \text{heis}(V) \longrightarrow V \longrightarrow 0.$$

Writing $\mathbb{R} \rtimes V$ with V abelian gives a split algebra and loses the nonzero bracket (7.4). A suitable notation is $\mathbb{R} \oplus_{\omega} V$. Accordingly, (7.7) should be written

$$\text{sch}(n-2) \simeq \text{heis}(\mathbb{R}^2 \otimes \mathbb{R}^{n-2}) \rtimes (\mathfrak{sl}_2 \oplus \mathfrak{so}(n-2)).$$

10. The claimed Cartan algebra in §7.2 is not commutative (p. 89)

The paper calls the subalgebra spanned by the usual Cartan algebra, M , and $B_{-m-1, m}$ “maximal commutative.” But its own equation (7.5c) states

$$[M, B_{-m-1, m}] = 2B_{-m-1, m} \neq 0.$$

This is an immediate contradiction. The likely intended additional generator is the central element $B_{m+1, m}$, not $B_{-m-1, m}$.

11. Section 7.2 does not span the odd-dimensional case as claimed

For odd n , the root list in item (2) restricts j to $|j| = 1, \dots, m-1$ and omits the $j = 0$ root operators. Yet the surrounding construction explicitly includes odd dimensions. The displayed list spans only the even-coordinate part unless the missing $B_{m,0}$ and $B_{m+1,0}$ terms are added.

12. Incorrect real-form identification $\mathfrak{sl}(2) \simeq \mathfrak{so}(3)$

In the real split setting used by the paper,

$$\mathfrak{sl}(2, \mathbb{R}) \simeq \mathfrak{so}(2, 1),$$

not the compact algebra $\mathfrak{so}(3)$. The statement is correct only after complexification:

$$\mathfrak{sl}(2, \mathbb{C}) \simeq \mathfrak{so}(3, \mathbb{C}).$$

The same qualification is needed when $\mathfrak{so}_{23}(3)$ is introduced later.

13. Equations (8.36a,b): the Casimir shifts are reversed (p. 103)

From (8.34),

$$[B_{-3,2}, B_{3,-2}] = -M, \quad [M, B_{-3,2}] = 2B_{-3,2}.$$

The central quadratic element is

$$\mathcal{C}_{23} = 4B_{3,-2}B_{-3,2} - (M+1)^2 + 1 = 4B_{-3,2}B_{3,-2} - (M-1)^2 + 1.$$

The paper prints $M-1$ in the first expression and $M+1$ in the second. Direct commutation of the printed expression with $B_{-3,2}$ gives a nonzero result. Equation (9.33) later prints the correct ordering, so the paper is internally inconsistent.

14. Equations (8.38a,b) and (8.39e,f) inherit the wrong Casimir shifts

The corrected (8.38) is

$$\begin{aligned} (2z_{-2}z_2 + 2z_{-3}z_3)\Delta_6^\diamond &= 4B_{2,-3}B_{-2,3} - (N_1 + M - 1)(-N_1 + M - 1) \\ &= 4B_{-2,3}B_{2,-3} - (N_1 + M + 1)(-N_1 + M + 1). \end{aligned}$$

Accordingly, the corrected last two heat factorizations are

$$\begin{aligned} 2y_{-1}y_1\mathcal{L}_2 &= -4B_{2,-3}B_{-2,3} - (N_1 + M - 1)(N_1 - M + 1), \\ &= -4B_{-2,3}B_{2,-3} - (N_1 + M + 1)(N_1 - M - 1). \end{aligned}$$

Direct substitution of the differential operators in (8.8)-(8.9) confirms these corrections. The four factorizations (8.39a-d) are correct. The six reduced confluent factorizations in §8.8 also compose correctly despite the bad intermediate Casimir display.

15. Proposition 9.1 is overstated (p. 122)

Proposition 9.1 asserts both (9.40a) and (9.40b) without assumptions on the contours. Its proof explicitly admits:

If the contours satisfy the requirements of Prop. 7.1, then ... (9.40b).

Thus the M -eigenvalue equations (9.40b) are not established, and are generally not true, for arbitrary contours. The proposition must include the endpoint, convergence, branch, and differentiation conditions required by Proposition 7.1. The proof also cites only (9.42b) where it needs both (9.42a) and (9.42b).

16. Theorem 10.1(ii) has the wrong endpoint condition (p. 132)

Set

$$q = t^2 - w, \quad g(t, w) = e^{2t} q^{-c+1/2}, \quad \mathcal{F}(c) = w\partial_w^2 + c\partial_w - 1.$$

Direct differentiation gives

$$\mathcal{F}(c)g = \partial_t \left[\frac{1}{4} e^{2t} q^{-c-1/2} ((1-2c)t - 2q) \right].$$

The theorem instead assumes the vanishing of

$$e^{2t} q^{-c+3/2} \Big|_{\gamma(0)}^{\gamma(1)},$$

which is not the boundary term above and does not imply its vanishing. A sufficient fixed-contour endpoint condition is

$$e^{2t} (t^2 - w)^{-c-1/2} ((1-2c)t - 2(t^2 - w)) \Big|_{\gamma(0)}^{\gamma(1)} = 0.$$

If the contour or its endpoints depend on w , the resulting endpoint derivatives must additionally be included.

Parts (i) and (iii) of Theorem 10.1 pass the direct total-derivative check.

17. Wrong $0F_1$ index in §10.7 (p. 129)

The indices of

$$\mathcal{F}_\alpha = wD^2 + (1 + \alpha)D - 1$$

at $w = 0$ are 0 and $-\alpha$. Since $\alpha = c - 1$, the second index is

$$1 - c = -\alpha,$$

not " $\alpha = 1 - c$ " as printed.

II. Definite notation and transcription errors

These usually do not invalidate the surrounding argument, but they can cause incorrect reuse.

Location	Printed	Correction
§2.2, p. 14	“For $a \in \mathbb{C}$ and $n \in \mathbb{N}$ ” before defining $(a)_j$	$j \in \mathbb{N}$; also state $(a)_0 = 1$
(3.12a), p. 22 after (4.13), p. 38	N_i followed by $j = 1, \dots, m$ representations acting on $\Lambda_+^\eta(\mathcal{Y})$	$i = 1, \dots, m$ $\Lambda_+^\eta(\mathcal{V})$
(4.25), p. 42	$B^{\diamond, \eta}, \alpha^{\diamond, \eta}$ on the chosen section	the newly defined $B^{\gamma, \eta}, \alpha^{\gamma, \eta}$ notation
p. 46	$\Psi K(y) = K(y, 1, 0)$ in coordinates (y, R, p)	$K(y, 0, 1)$, since the null cone is $R = 0$ and the section is $p = 1$
§7.3 heading, p. 90	“sch($n + 2$) in n dimensions”	sch($n - 2$)
§8.7, p. 102	subalgebras in sch(5), flip in Sch(5)	sch(2), Sch(2)
(8.50b), p. 108	exponent $(\alpha + \rho - 1)/2$	$(\alpha + \theta - 1)/2$
(8.53d), p. 110	“ xs Re”	Re
§9.8, p. 118	“subalgebras of sch(2)”	sch(1)
§9.8, p. 119	sch(6), sch(5)	either ambient $\mathfrak{so}(6), \mathfrak{so}(5)$, or Schrödinger sch(2), sch(1); the printed labels are impossible in context
p. 120	“ B, N_1, M ” should carry superscripts	there is no N_1 in this sch(1) calculation; use B, M
(10.16a), p. 127	$u \partial_u$	$u_1 \partial_{u_1}$
(10.20), p. 128	$v = y_- y_+ / 2, u = y_+$	$v = x_- x_+ / 2, u = x_+$, using the variables defined in (10.18)

III. Missing hypotheses and overstatements

1. Complex powers are not global smooth functions on the stated real spaces

Equations such as (3.25), Lemma 5.1, Lemma 6.1, and the wave-packet kernels use arbitrary complex powers and then call the results functions “on $\mathbb{R}^n$ ” or “harmonic on $\mathbb{R}^n$.” For nonintegral exponents these are only locally defined after:

- removing the zero hyperplanes of the bases;
- choosing branches on simply connected components; or
- passing to an appropriate covering/Riemann surface.

The harmonic identities are locally correct. The global statements are false without this qualification.

2. Proposition 3.1 needs regularity

“Let f be a function” is too weak. To apply the Laplacian classically, f must be at least C^2 (or holomorphic in the complex setting) on the relevant domain.

3. Integral theorems need more than endpoint vanishing

This affects Proposition 3.2; Propositions 5.2, 6.2, 7.1, 8.1, and 9.1; and Theorems 5.4, 6.3, 8.2, 9.2, and 10.1.

The displayed endpoint equality is not, by itself, sufficient. The statements also need:

- a contour avoiding non-endpoint singularities;
- a coherent branch on the lifted contour;
- convergence (or a specified regularization);
- permission to differentiate under the integral sign;
- treatment of any w -dependent contour or endpoints.

The algebraic total-derivative identities in Theorems 5.4, 6.3, 8.2, and 9.2 are correct. The issue is the theorem-level analytic sufficiency of their hypotheses. Theorem 10.1(ii) has the stronger algebraic defect described above.

4. Equation (2.3) requires analytic-continuation language

For $c_j \in \{0, -1, -2, \dots\}$, the quotient

$$\frac{{}_kF_m}{\Gamma(c_1) \cdots \Gamma(c_m)}$$

is not a literal quotient of the originally defined ${}_kF_m$, because that original series excluded those denominator parameters. The second series in (2.3) gives the continuation/regularized definition. The equality should be stated first away from gamma poles and then continued analytically.

The regularization does not change the convergence class in w ; it does not turn a divergent ${}_pF_q$ series with $p > q + 1$ into a convergent one.

5. The early $2F_0$ limit is under-specified

In §2.3, the paper defines $2F_0$ by a $c \rightarrow \infty$ limit without specifying the approach sector. The later §8.10 correctly imposes a sector on c . Without such a prescription, Stokes behavior makes the early definition incomplete.

6. Noninteger homogeneity and Weyl formulas need branch/sheet restrictions

The assertion after (4.27) that “we do not have problems in the complex case” is too strong. For noninteger complex homogeneity, global formulas such as (4.36b) and (4.38b) contain

$$\left(-\frac{\langle y, y \rangle}{2}\right)^\eta, \quad y_{-j}^\eta,$$

and require a branch, a chosen sheet, or a covering/density-bundle formulation. Positive homogeneity alone only defines scaling by $t > 0$.

7. “Weyl group of $O(2m)$ ” is nonstandard terminology

The group in §3.9 is the full signed-permutation group

$$\mathbb{Z}_2^m \rtimes S_m.$$

That is natural for the disconnected group $O(2m)$, but the usual Weyl group of the connected Lie algebra/group $SO(2m)$ is the even signed-permutation group

$$\mathbb{Z}_2^{m-1} \rtimes S_m.$$

The paper defines its convention, so this is not an internal algebraic error, but readers should not identify it with the standard type- D_m Weyl group without qualification.

IV. Section-by-section equation ledger

“Pass” below means no additional algebraic contradiction was found beyond the listed caveats. It does not remove the global branch and convergence qualifications in Section III.

Section	Distinct labels	Audit result
§2	18	Differential equation (2.4) and classification (2.7)-(2.15) pass. Equation (2.3) and the early $2F_0$ definition need analytic-continuation/sector qualifications.
§3	53	Lie-algebra, Laplacian, Mellin, and dimensional-reduction identities pass. Typo in (3.12a); Propositions 3.1-3.2 need regularity/convergence hypotheses.
§4	73	Theorem 4.2 and Corollaries 4.3-4.4 pass. Equations (4.7)-(4.8) have reversed/non-single-valued maps; (4.25) is notationally inconsistent; p. 46 has the R, p swap. Foundational pullback/Lie-derivative definitions are wrong.
§5	133	Core operator (5.15), parameter map, Casimirs (5.20), reductions (5.21)-(5.23), standard solutions, integral differential identity, and connection matrix (5.45)-(5.47) pass. Exceptions: two transmutation signs, four p. 60 factorizations, and condition (5.29).
§6	100	Coordinate reduction, quadratic transformation, standard solutions, recurrence relations, and Theorem 6.3’s formal identities pass. Exceptions: three transmutation lines and two p. 81 scalar signs.
§7	86	Differential realizations and generalized heat symmetries pass. Structural description fails at (7.6)-(7.7), omits odd roots, misidentifies the real $\mathfrak{sl}_2$ form, and names a noncommutative algebra “commutative.”

Section	Distinct labels	Audit result
§8	112	Confluent operator, parameter map, all six transmutations, all six reduced factorizations, recurrence relations, and Theorem 8.2's formal identities pass. Exceptions: (8.36), (8.38), (8.39e,f), and typo (8.50b).
§9	86	Quadratic transformation, four transmutations, Casimir (9.33), heat/Hermite factorizations, recurrences, and Theorem 9.2's formal identities pass. Proposition 9.1 is missing its contour hypothesis; several algebra labels are wrong.
§10	57	Reduction, Kummer transformation, transmutations, factorizations, recurrence relations, asymptotics, and connection formulas pass. Exceptions: notation in (10.16a)/(10.20), the sign of the second index in §10.7, and Theorem 10.1(ii).

V. What remains valid

The following major claims survived the audit:

- the reduction of the ambient Laplacian to the hypergeometric, Gegenbauer, confluent, Hermite, and ${}_0F_1$ operators;
- the principal parameter identifications;
- Theorem 4.2 and its conformal-Laplacian consequences;
- the formal total-derivative identities behind Theorems 5.4, 6.3, 8.2, and 9.2;
- the standard hypergeometric and confluent connection formulas at generic parameters;
- all confluent, Hermite, and ${}_0F_1$ transmutation relations;
- all Hermite, confluent, and ${}_0F_1$ one-variable factorizations.

The paper can therefore be repaired without changing its main conceptual thesis, but the printed formulas should not be used as a reference until the corrections in Sections I-III are applied.

Bottom line

This is a valuable and largely coherent work with a nontrivial number of correct derivations. It nevertheless contains enough false identities and theorem-level omissions that the phrase “there are some corrections” in the introduction understates the remaining problems. The errors are not merely cosmetic: several displayed intertwiners, six factorization/Casimir consequences, one proposition hypothesis, and one integral theorem condition are mathematically wrong as printed.