

Superconductivity, superfluidity and Bose-Einstein condensation 1102-6'BEC

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Rules of passing

From each set of problems chose one and type complete answers with all explanations, details, literature and AI prompts and replies, in a form of an essay. Essays can be single, double or triple authors maximally. One version with all contributing authors should be send as pdf by e-mail on my address byczuk@fuw.edu.pl. Final grade will be proportional to the number of submitted essays and their qualities. To get 5! an oral exam is additionally required.

**Lectures are on Thursdays at
11:15-12:00 in A2.11 room.**

1 Preliminary Program

1. Quantum coherence on macroscopic scale
 - (a) Introduction
 - (b) Brief historical notes
 - (c) Nobel prizes
2. Bose-Einstein condensation
 - (a) Standard description of Bose-Einstein condensation (BEC)
 - (b) Symmetry and phase transitions
 - (c) Off diagonal long range order in BEC
 - (d) Coherent states in harmonic oscillator
 - (e) BEC as a coherent state
3. Superfluidity
 - (a) Classical vs. quantum fluids
 - (b) Superfluidity experimentally
 - (c) Two component fluid model (Tisza)
 - (d) Probability current density and quantum mechanics on macroscopic scale
 - (e) Two fluid (hydro) thermodynamics (Tisza and Landau)

- (f) Quasiparticles and superfluidity (Landau)
- (g) Bogoliubov theory of collective excitations in weakly interacting Bose gas

4. Superconductivity

- (a) Superconducting phenomena
- (b) Electrodynamics of superconductors
- (c) Rigidity of superconducting state
- (d) Thermodynamics of superconductors - condensation energy
- (e) Ginzburg-Landau theory of uniform superconductors
- (f) Ginzburg-Landau theory of inhomogeneous superconductors
- (g) Ginzburg-Landau theory of superconductors in a magnetic field
- (h) Izotop effect and attractive interaction among electrons
- (i) BCS ground state
- (j) Analysis of the BCS ground state wave function
- (k) BCS mean-field theory

2 Problem sets

1. Set one - BEC, deadline May 1st, 2026

- (a) Describe methods of trapping and cooling of alkali atom gases and the methods of observing the BEC.
- (b) describe an example how changing a model one can get type I, II and III BEC.
- (c) Describe BEC as a symmetry breaking phase transition in d-dimensional space with the dispersion relation

$$\epsilon_{\mathbf{k}} = \epsilon_0 \left| \frac{\mathbf{k}}{k_0} \right|^s,$$

with ϵ_0 energy constant, k_0 wave number constant. Find transition temperature T_c , heat capacity at constant volume C_V , pressure p , entropy S , and internal energy U as a function of temperature T .

2. Set two - Superfluidity, deadline June 1st, 2026

- (a) Discuss in details the Feynman theory of HeII and his derivation of the roton spectrum. Find thermodynamics of the system made both of phonons (linear dispersion) and rotons (gapped quadratic dispersion) excitations at different $\mathbf{k}$.
- (b) Discuss mathematical rigorousness of the Bogoliubov substitution $\hat{a}_0 \rightarrow \langle \hat{a}_0 \rangle = a_0 \in C$.
- (c) Discuss in details derivation and physics of time independent and time dependent Gross-Pitaevskii equations.

- (d) Derive from Landau's superfluid hydrodynamics the wave equations for the first and the second sound. Express microscopically the sound velocities and estimate their values. describe how the second sound was detected experimentally.
3. Set three - Superconductivity, deadline July 1st, 2026

- (a) (i) Solving the Maxwell-London equations

$$\nabla^2 \mathbf{B} = \frac{1}{\lambda^2} \mathbf{B}, \quad \nabla \cdot \mathbf{B} = 0, \quad \text{for } r < R$$

and

$$\nabla^2 \mathbf{B} = 0, \quad \nabla \cdot \mathbf{B} = 0, \quad \text{for } r > R$$

for a spherical superconductor of radius R find the distribution of the magnetic induction $\mathbf{B}$ in space. Take the boundary conditions at infinity $\mathbf{B} \rightarrow B_0 \mathbf{e}_z$.

(ii) A London vortex in a superconductor can be modeled as having a cylindrical core of normal metal of radius ξ_0 . Find distribution of the magnetic induction in cylindrical coordinates (r, ϕ, z) for $\xi_0 < r \ll \lambda$ and for $r > \lambda$. Find the corresponding current when $\xi_0 < r \ll \lambda$.

λ is a penetration depth in both problems.

- (b) Within the linearized Ginzburg-Landau theory describe superconductor in a strong magnetic field. Derive conditions on type one and type two superconductors. Analyze Abrikosov vortex lattice and show that they form a hexagonal structure.
- (c) Within the Ginzburg-Landau theory describe in detail the Anderson-Higgs mechanism in superconductors. Show how the photons get the mass and how Goldstone mode is absorbed into the vector potential. Review briefly experiments on Higgs physics in superconductors.
- (d) Solve in detail a problem of the Cooper pair. Derive equation for energy of a moving pair of electrons on the background of non-interacting Fermi sea. What is the binding energy? Calculate the average radius square of the pair. Model and plot the shape of the wave function in one dimension.
- (e) On the base of the mean-field BCS theory derive equations for the density of states in superconductors, ultrasonic attenuation and nuclear relaxation.

- L.D. Landau and E.M. Lifshyc *Hydrodynamika*
- M. Sigrist *Statistical Physics* Lecture notes, <https://edu.itp.phys.ethz.ch/hs14/StatPhys/LectureNotes.pdf>
- S.H. Simone, *Lecture Notes on Quantum Matter*, <https://www-thphys.physics.ox.ac.uk/people/SteveSimon/QCM2022/QuantumMatter.pdf>
- C. Timm, *Theory of superconductivity*, https://tu-dresden.de/mn/physik/itp/cmt/ressourcen/dateien/skripte/Skript_Supra.pdf?lang=de
- J.R. Schrieffer, *Theory of superconductivity*
- M. Tinkham, *Introduction to superconductivity*
- M. Cyrot, D. Pavuna, *Wstęp do nadprzewodnictwa*

3 Literature

- J.F. Annett *Superconductivity, Superfluids and Condensates*
- R. P. Feynman *Wykłady z mechaniki statystycznej*