

Implications of G00Fy for models of electroweak interactions

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based on

- P.M. Ferreira, B.G., O.M. Ogreid, and P. Osland, "New Symmetries of the Two-Higgs-Doublet Model", *Eur. Phys. J. C* **84** (2024) 3, 234,
- P.M. Ferreira, B.G. and O.M. Ogreid, "Imaginary scaling invariance of the one-loop effective potential", *Phys. Rev. D* **113** (2026) 7, 075036,
- B.G., O.M. Ogreid, "GOOFy - a systematic approach", *JHEP* **07** (2026) 038.

Global symmetries of the 2HDM

- spontaneously broken $SU(2)_L \times U(1)_Y$ gauge symmetry
- scalar fields: $\Phi_1, \Phi_2 \sim (2, \frac{1}{2})$

$$\begin{aligned} V = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right) + \\ & \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \\ & \left\{ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\}, \end{aligned}$$

where m_{12}^2 and $\lambda_{5,6,7}$ might be complex.

Global symmetries of the 2HDM

- *Higgs-family symmetries*, unitary transformations mix both doublets,

$$\Phi_i \rightarrow \Phi'_i = \sum_{j=1}^2 U_{ij} \Phi_j, \quad U \in U(2)$$

e.g. Z_2 :

$$\Phi_1 \rightarrow \Phi_1, \quad \Phi_2 \rightarrow -\Phi_2,$$

prevents the occurrence of tree-level flavour-changing neutral currents (FCNC).

- *generalized CP (GCP)*, transformations:

$$\Phi_i \rightarrow \Phi'_i = \sum_{j=1}^2 X_{ij} \Phi_j^*, \quad X \in U(2)$$

e.g. "standard" CP transformation (CP1):

$$\Phi_i \rightarrow \Phi_i^*$$

Global symmetries of the 2HDM

S	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
CP1			real					real	real	real
Z_2			0						0	0
U(1)			0					0	0	0
CP2		m_{11}^2	0		λ_1					$-\lambda_6$
CP3		m_{11}^2	0		λ_1			λ_{134}	0	0
SO(3)		m_{11}^2	0		λ_1		$\lambda_1 - \lambda_3$	0	0	0

Table 1: Relations between 2HDM scalar potential parameters for each of the six symmetries discussed, $\lambda_{134} \equiv \lambda_1 - \lambda_3 - \lambda_4$.

$$\begin{aligned}
 V = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - (m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}) + \\
 & \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \\
 & \left\{ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\},
 \end{aligned}$$

Running 2HDM parameters and RG stable relations

The 1-loop β -functions for the quadratic couplings

$$\beta_{m_{11}^2} = 3\lambda_1 m_{11}^2 + (2\lambda_3 + \lambda_4) m_{22}^2 - 3 (\lambda_6^* m_{12}^2 + \text{h.c.}) - \frac{1}{4} (9g^2 + 3g'^2) m_{11}^2 + \beta_{m_{11}^2}^F,$$

$$\beta_{m_{22}^2} = (2\lambda_3 + \lambda_4) m_{11}^2 + 3\lambda_2 m_{22}^2 - 3 (\lambda_7^* m_{12}^2 + \text{h.c.}) - \frac{1}{4} (9g^2 + 3g'^2) m_{22}^2 + \beta_{m_{22}^2}^F,$$

$$\beta_{m_{12}^2} = -3 (\lambda_6 m_{11}^2 + \lambda_7 m_{22}^2) + (\lambda_3 + 2\lambda_4) m_{12}^2 + 3\lambda_5 m_{12}^{2*} - \frac{1}{4} (9g^2 + 3g'^2) m_{12}^2 + \beta_{m_{12}^2}^F,$$

Running 2HDM parameters and RG stable relations

and 1-loop β functions for the quartic ones,

$$\begin{aligned}
 \beta_{\lambda_1} &= 6\lambda_1^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + |\lambda_5|^2 + 12|\lambda_6|^2 \\
 &\quad + \frac{3}{8}(3g^4 + g'^4 + 2g^2g'^2) - \frac{3}{2}\lambda_1(3g^2 + g'^2) + \beta_{\lambda_1}^F, \\
 \beta_{\lambda_2} &= 6\lambda_2^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + |\lambda_5|^2 + 12|\lambda_7|^2 \\
 &\quad + \frac{3}{8}(3g^4 + g'^4 + 2g^2g'^2) - \frac{3}{2}\lambda_2(3g^2 + g'^2) + \beta_{\lambda_2}^F, \\
 \beta_{\lambda_3} &= (\lambda_1 + \lambda_2)(3\lambda_3 + \lambda_4) + 2\lambda_3^2 + \lambda_4^2 + |\lambda_5|^2 + 2(|\lambda_6|^2 + |\lambda_7|^2) + 8\text{Re}(\lambda_6\lambda_7^*) \\
 &\quad + \frac{3}{8}(3g^4 + g'^4 - 2g^2g'^2) - \frac{3}{2}\lambda_3(3g^2 + g'^2) + \beta_{\lambda_3}^F, \\
 \beta_{\lambda_4} &= (\lambda_1 + \lambda_2)\lambda_4 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 4|\lambda_5|^2 + 5(|\lambda_6|^2 + |\lambda_7|^2) + 2\text{Re}(\lambda_6\lambda_7^*) \\
 &\quad + \frac{3}{2}g^2g'^2 - \frac{3}{2}\lambda_4(3g^2 + g'^2) + \beta_{\lambda_4}^F, \\
 \beta_{\lambda_5} &= (\lambda_1 + \lambda_2 + 4\lambda_3 + 6\lambda_4)\lambda_5 + 5(\lambda_6^2 + \lambda_7^2) + 2\lambda_6\lambda_7 \\
 &\quad - \frac{3}{2}\lambda_5(3g^2 + g'^2) + \beta_{\lambda_5}^F, \\
 \beta_{\lambda_6} &= (6\lambda_1 + 3\lambda_3 + 4\lambda_4)\lambda_6 + (3\lambda_3 + 2\lambda_4)\lambda_7 + 5\lambda_5\lambda_6^* + \lambda_5\lambda_7^* \\
 &\quad - \frac{3}{2}\lambda_6(3g^2 + g'^2) + \beta_{\lambda_6}^F, \\
 \beta_{\lambda_7} &= (6\lambda_2 + 3\lambda_3 + 4\lambda_4)\lambda_7 + (3\lambda_3 + 2\lambda_4)\lambda_6 + 5\lambda_5\lambda_7^* + \lambda_5\lambda_6^* \\
 &\quad - \frac{3}{2}\lambda_7(3g^2 + g'^2) + \beta_{\lambda_7}^F,
 \end{aligned}$$

where the β_x^F terms contain all contributions coming from fermions.

Running 2HDM parameters and RG stable relations

$$Z_2 \text{ symmetry} \Rightarrow m_{12}^2 = 0, \lambda_6 = \lambda_7 = 0 \Rightarrow \beta_{m_{12}^2} = 0, \beta_{\lambda_6} = \beta_{\lambda_7} = 0$$

a symmetry-based conditions on m^2 's and λ 's are preserved by RG running at the one-loop order.

For the Z_2 -symmetric model

$$\beta_{\lambda_5} = \left[\lambda_1 + \lambda_2 + 4\lambda_3 + 6\lambda_4 - \frac{3}{2} (3g^2 + g'^2) \right] \lambda_5$$

$\lambda_5 = 0$ is a *fixed point* of this RG equation: if at any scale $\lambda_5 = 0$, that coupling will remain zero for all renormalization scales. **Such fixed points of RG equations are usually fingerprints of symmetries**, and indeed that is the case here: if $m_{12}^2 = 0$ and $\lambda_6 = \lambda_7 = 0$, the extra constraint $\lambda_5 = 0$ takes us from Z_2 -symmetric model to $U(1)$ -symmetric.

Running 2HDM parameters and RG stable relations

We have noticed that

$$\{m_{11}^2 + m_{22}^2 = 0, \lambda_1 - \lambda_2 = 0, \lambda_6 + \lambda_7 = 0\}$$

- constitutes a fixed point of the 1-loop RG equations,
- are basis transformation invariants.

$$\begin{aligned}\beta_{m_{11}^2 + m_{22}^2} &= 3(\lambda_1 m_{11}^2 + \lambda_2 m_{22}^2) + (2\lambda_3 + \lambda_4)(m_{11}^2 + m_{22}^2) \\ &\quad - 3[(\lambda_6^* + \lambda_7^*)m_{12}^2 + \text{h.c.}] - \frac{1}{4}(9g^2 + 3g'^2)(m_{11}^2 + m_{22}^2) \\ \beta_{\lambda_1 - \lambda_2} &= 6(\lambda_1^2 - \lambda_2^2) + 12(|\lambda_6|^2 - |\lambda_7|^2) - \frac{3}{2}(\lambda_1 - \lambda_2)(3g^2 + g'^2) \\ \beta_{\lambda_6 + \lambda_7} &= 6(\lambda_1\lambda_6 + \lambda_2\lambda_7) + (3\lambda_3 + 2\lambda_4)(\lambda_6 + \lambda_7) + 6\lambda_5(\lambda_6^* + \lambda_7^*) \\ &\quad - \frac{3}{2}(\lambda_6 + \lambda_7)(3g^2 + g'^2)\end{aligned}$$

It turns out that

$$\{m_{11}^2 + m_{22}^2 = 0, \lambda_1 - \lambda_2 = 0, \lambda_6 + \lambda_7 = 0\}$$

is also the 2-loop fixed point.



Conclusion:

Perhaps there is a symmetry behind the fixed point:

$$\{m_{11}^2 + m_{22}^2 = 0, \lambda_1 - \lambda_2 = 0, \lambda_6 + \lambda_7 = 0\}$$

The G00Fy symmetry: bosons

$$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix},$$

The transformation

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{pmatrix}$$

implies

$$m_{11}^2 + m_{22}^2 = 0, \quad \lambda_1 - \lambda_2 = 0, \quad \lambda_6 + \lambda_7 = 0$$

The G00Fy symmetry: bosons

$$\begin{aligned}\Phi_1 &\rightarrow \Phi'_1 = -\Phi_2^*, & \Phi_1^\dagger &\rightarrow \Phi_2^T \neq (\Phi'_1)^\dagger, \\ \Phi_2 &\rightarrow \Phi'_2 = \Phi_1^*, & \Phi_2^\dagger &\rightarrow -\Phi_1^T \neq (\Phi'_2)^\dagger.\end{aligned}$$

- Higgs kinetic terms

$$\mathcal{L}_k = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2),$$

where

$$D^\mu = \partial^\mu + \frac{ig}{2} \sigma_i W_i^\mu + i \frac{g'}{2} B^\mu,$$

$\mathcal{L}_k$ remains invariant if the above transformation of $\Phi_{1,2}$ is supplemented by

$$\begin{aligned}\partial_\mu &\rightarrow -i\partial_\mu, & (x^\mu &\rightarrow ix^\mu), \\ B_\mu &\rightarrow iB_\mu, \\ W_{1\mu} &\rightarrow iW_{1\mu}, & W_{2\mu} &\rightarrow -iW_{2\mu}, & W_{3\mu} &\rightarrow iW_{3\mu}.\end{aligned}$$

The G00Fy symmetry: bosons

- Gauge kinetic terms

$$\mathcal{L}^B = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{i\mu\nu}W_i^{\mu\nu},$$

where $B^{\mu\nu} = \partial^\nu B^\mu - \partial^\mu B^\nu$ and $W_i^{\mu\nu} = \partial^\nu W_i^\mu - \partial^\mu W_i^\nu + g\epsilon_{ijk}W_j^\mu W_k^\nu$.

Under r_0 transformation

$$B^{\mu\nu} \rightarrow B^{\mu\nu}, \\ W_1^{\mu\nu} \rightarrow W_1^{\mu\nu}, \quad W_2^{\mu\nu} \rightarrow -W_2^{\mu\nu}, \quad W_3^{\mu\nu} \rightarrow W_3^{\mu\nu}$$

The GOFy symmetry: bosons

We shall allow for transformations of the following form,

$$x^\mu \rightarrow i x^\mu, \quad \Phi \rightarrow X_\phi \Phi^*, \quad \Phi^\dagger \rightarrow \Phi^T \bar{X}_\phi^\dagger,$$

where X_ϕ and $\bar{X}_\phi$ are both unitary $N_\phi \times N_\phi$ matrices. Requiring

$$\mathcal{L}_{\text{kin}} = (\partial_\mu \Phi^\dagger) (\partial^\mu \Phi)$$

to be invariant under the above transformation implies $\bar{X}_\phi = -X_\phi$.
Then

$$x^\mu \rightarrow i x^\mu, \quad \Phi \rightarrow X_\phi \Phi^*, \quad \Phi^\dagger \rightarrow -\Phi^T X_\phi^\dagger$$

will constitute the GOFy transformation.

The GOOFy symmetry: bosons

Scalar potential

$$V(\Phi) = \Phi^\dagger M^2 \Phi$$

Hermiticity implies that M^2 is a Hermitian $N_\phi \times N_\phi$ matrix.
Goofy-invariance of the $\Phi^\dagger \Phi$ mass terms requires

$$M^2 = -X_\phi^T (M^2)^T X_\phi^*$$

Therefore

$$\text{Tr } M^2 = 0$$

This requirement forbids $\Phi^\dagger M^2 \Phi$ mass term in the case of a single complex scalar field ($N_\phi = 1$). For $N_\phi \geq 2$, this type of mass terms is allowed.

The G00Fy symmetry: bosons

Example: $N_\phi = 2$

$$\Phi \rightarrow X_\phi \Phi^*, \quad \Phi^\dagger \rightarrow -\Phi^T X_\phi^\dagger$$

Then using the mass-term invariance condition

$$M^2 = -X_\phi^T (M^2)^T X_\phi^*$$

one finds in the real and diagonal basis (where $m_{12}^2 = 0$)

$$X_\phi = \begin{pmatrix} 0 & e^{i\theta_1} \\ e^{i\theta_2} & 0 \end{pmatrix}$$

and the $\Phi^\dagger \Phi$ mass matrix is of the form.

$$M^2 = \begin{pmatrix} m_{11}^2 & 0 \\ 0 & -m_{11}^2 \end{pmatrix}$$

The GOFy symmetry: fermions

For fermions we also start with C-inspired transformation:

$$x^\mu \rightarrow ix^\mu, \quad \Psi \rightarrow -X_\psi \gamma_0 C \Psi^*, \quad \bar{\Psi} \rightarrow -\Psi^T C^{-1} \bar{X}_\psi^\dagger \quad (2)$$

where $\Psi^T \equiv (\psi_1, \dots, \psi_{N_\psi})$ denotes a multiplet of N_ψ fermionic fields while X_ψ and $\bar{X}_\psi$ are unitary $N_\psi \times N_\psi$ matrices. Again, we allow for independent field transformations of ψ and $\bar{\psi}$. As before GOFy symmetries are defined by demanding invariance of

$$\mathcal{L}_{\text{kin}} = i\bar{\Psi}\gamma^\mu\partial_\mu\Psi$$

under (2), which in turn implies $\bar{X}_\psi = -iX_\psi$ i.e.

$$x^\mu \rightarrow ix^\mu, \quad \Psi \rightarrow -X_\psi \gamma_0 C \Psi^*, \quad \bar{\Psi} \rightarrow -\Psi^T C^{-1} iX_\psi^\dagger$$

which will therefore constitute the GOFy transformation for fermionic fields.

The G00Fy symmetry: fermions with Dirac mass

In order to preserve the Dirac fermionic mass term $\mathcal{L}_{\text{D-mass}} = -\bar{\Psi} M_D \Psi$ (with $M_D^\dagger = M_D$) under the G00Fy transformation

$$\Psi \rightarrow -X_\psi \gamma_0 C \Psi^*, \quad \bar{\Psi} \rightarrow -\Psi^T C^{-1} i X_\psi^\dagger,$$

the following condition must hold

$$M_D = i X_\psi^T M_D^* X_\psi^*$$

Since M_D is Hermitian, it turns out, that the above equation implies $M_D = 0$ regardless of N_ψ for any unitary X_ψ . We conclude that fermionic Dirac mass terms are not compatible with G00Fy symmetry.

Yukawa interactions

We are going to consider the following Yukawa interactions:

$$-\mathcal{L}_Y = \bar{\Psi}\Gamma\Phi\Psi + \bar{\Psi}\Phi^\dagger\Gamma^\dagger\Psi \equiv \bar{\psi}_a(\Gamma_i)_{ab}\phi_i\psi_b + \bar{\psi}_a(\phi_i)^*(\Gamma_i)_{ba}^*\psi_b.$$

Requiring invariance of $\mathcal{L}_Y$ under G00Fy transformation

$$\begin{aligned}\Psi &\rightarrow -X_\psi\gamma_0 C\Psi^*, & \bar{\Psi} &\rightarrow -\Psi^T C^{-1}iX_\psi^\dagger, \\ \Phi &\rightarrow X_\phi\Phi^*, & \Phi^\dagger &\rightarrow -\Phi^T X_\phi^\dagger,\end{aligned}$$

implies the following Yukawa-invariant condition:

$$\Gamma_i^* = i X_\psi^T \Gamma_j X_\psi^* (X_\phi)_{ji}$$

- X_ψ and X_ϕ are fixed,
- system of linear and homogeneous equations for $(\Gamma_i)_{ab}$,
- determinant of the coefficient matrix of the system of equations must vanish.

Yukawa interactions

Examples:

The generic condition for invariance of Yukawa couplings:

$$\Gamma_i^* = i X_\psi^T \Gamma_j X_\psi^* (X_\phi)_{ji}$$

For $N_\phi = 2$ and $N_\psi = 2$

$$X_\phi = \begin{pmatrix} 0 & e^{i\theta_1} \\ e^{i\theta_2} & 0 \end{pmatrix}$$

one finds

$$\Gamma_1^* = ie^{i\theta_2} X_\psi^T \Gamma_2 X_\psi^*, \quad \Gamma_2^* = ie^{i\theta_1} X_\psi^T \Gamma_1 X_\psi^* \quad (3)$$

The above equations could be solved for $\Gamma_{1,2}$, e.g. first one finds an equation for Γ_1

$$\Gamma_1 = e^{-i(\theta_2 - \theta_1)} X_\psi^\dagger X_\psi^T \Gamma_1 X_\psi^* X_\psi, \quad (4)$$

which then allows to find a solution for Γ_2 .

Yukawa interactions - examples

We choose X_ψ to be a real orthogonal 2×2 matrix parameterized by an angle θ [1], i.e.

$$X_\psi = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \quad (5)$$

Non-trivial solutions exist whenever the determinant of the coefficient matrix of the resulting linear system of equations derived from (4) for entries of Γ_1 vanish, i.e.

$$64 [\cos(\theta_1 - \theta_2) - \cos(4\theta)]^2 \left[\sin \left(\frac{\theta_1 - \theta_2}{2} \right) \right]^4 = 0, \quad (6)$$

with solutions $\theta_2 = \theta_1$ or $\theta_2 = \theta_1 \pm 4\theta$.

[1] G. Ecker, W. Grimus and H. Neufeld, "A Standard Form for Generalized CP Transformations", J. Phys. A 20 (1987) L807.

We find six different non-trivial solutions of (4).

Solution 1:

$$X_\phi = e^{i\theta_1} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad X_\psi = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (7)$$

$$\Gamma_1 = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix}, \quad \Gamma_2 = -ie^{-i\theta_1} \begin{pmatrix} z_{11}^* & z_{12}^* \\ z_{21}^* & z_{22}^* \end{pmatrix}. \quad (8)$$

For all solutions, z_{ij} are arbitrary complex parameters.

Using the beta functions obtained by PyR@TE 3 [2], we verify that (3) are RG-stable up to two-loop order for X_ϕ , X_ψ and Γ_j of the form given in all six solutions.

Furthermore, we find that

$$M^2 = -X_\phi^T (M^2)^T X_\phi^* \quad \text{with} \quad M^2 = \begin{pmatrix} m_{11}^2 & 0 \\ 0 & -m_{11}^2 \end{pmatrix}. \quad (9)$$

is RG-stable up to two-loop order.

[2] L. Sartore and I. Schienbein, Comput. Phys. Commun. 261 (2021), 107819.

$$\text{Tr } M^2 = 0$$



Concluding, both the scalar potential and Yukawa couplings of the SM are inconsistent with the GOOFy symmetry.

Adopting the generic strategy (described and tested earlier in various contexts), the GOOFy transformation shall be:

$$\begin{array}{ll} \Phi & \rightarrow X_\phi \Phi^*, \\ q_L & \rightarrow -X_\alpha \gamma^0 C q_L^*, \\ d_R & \rightarrow -X_\beta \gamma^0 C d_R^*, \\ u_R & \rightarrow -X_\gamma \gamma^0 C u_R^*, \end{array} \quad \begin{array}{ll} \Phi^\dagger & \rightarrow -\Phi^T X_\phi^\dagger, \\ \bar{q}_L & \rightarrow -i q_L^T C^{-1} X_\alpha^\dagger, \\ \bar{d}_R & \rightarrow -i d_R^T C^{-1} X_\beta^\dagger, \\ \bar{u}_R & \rightarrow -i u_R^T C^{-1} X_\gamma^\dagger. \end{array}$$

The most general scalar potential for the 2HDM reads

$$\begin{aligned} V = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right) \\ & + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ & + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\ & + \left\{ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\} \end{aligned}$$

where, other than m_{12}^2 and $\lambda_{5,6,7}$, all parameters are real.

The generic Yukawa Lagrangian for this model is as follows

$$-\mathcal{L}_Y = \bar{q}_L \Gamma_a \Phi_a d_R + \bar{q}_L \Delta_a \tilde{\Phi}_a u_R + \bar{d}_R \Gamma_a^\dagger \Phi_a^\dagger q_L + \bar{u}_R \Delta_a^\dagger (\tilde{\Phi}_a)^\dagger q_L,$$

with $a = 1, 2$.

$$\Phi^\dagger M^2 \Phi - \text{invariance} \quad \Rightarrow \quad X_\phi = \begin{pmatrix} 0 & e^{i\theta_1} \\ e^{i\theta_2} & 0 \end{pmatrix}$$

$$\text{basis choice} \quad \Rightarrow \quad X_\psi = \begin{pmatrix} \cos \theta_\psi & \sin \theta_\psi & 0 \\ -\sin \theta_\psi & \cos \theta_\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

for $\psi = \alpha, \beta$ and γ .

Explicitly, the GOOFy transformations of the doublets read

$$\begin{aligned} \Phi_1 &\rightarrow e^{i\theta_1} \Phi_2^* & \Phi_1^\dagger &\rightarrow -e^{-i\theta_1} \Phi_2^T, \\ \Phi_2 &\rightarrow e^{i\theta_2} \Phi_1^* & \Phi_2^\dagger &\rightarrow -e^{-i\theta_2} \Phi_1^T. \end{aligned}$$

Strategy

- $\Gamma_1 = -iX_\alpha^T \Gamma_2^* X_\beta^* e^{-i\theta_2}, \quad \Gamma_2 = -iX_\alpha^T \Gamma_1^* X_\beta^* e^{-i\theta_1}. \quad (*)$
- Find the determinant (in terms of $\Delta\theta, \alpha, \beta, \gamma$) of the resulting coefficient matrix for $(\Gamma_i)_{ab}$ and solve for its zeros (constraints/solutions for $\Delta\theta, \alpha, \beta, \gamma$).
- For $\Delta\theta$ obtained above check invariance of the potential and find relations between parameters that must be satisfied for the invariance. For 2HDM, $\Delta\theta$'s obtained from determinants being zero allow for invariant potential.
- Adopt $\Delta\theta, \alpha, \beta, \gamma$ obtained above in the equations for Yukawa's (*) to determine final form of the Yukawa matrices.
- Check the 2-loop RGE stability of the invariance relations (*) (adopting Yukawa matrices that satisfy the invariance conditions).

The determinant D is then the product of the determinants of the four blocks,

$$D = D_1 D_2 D_3 D_4$$

where

$$\begin{aligned} D_1 &= 256 \sin^2 \left(\alpha + \beta - \frac{\Delta\theta}{2} \right) \sin^2 \left(\alpha - \beta + \frac{\Delta\theta}{2} \right) \\ &\quad \times \sin^2 \left(\alpha - \beta - \frac{\Delta\theta}{2} \right) \sin^2 \left(\alpha + \beta + \frac{\Delta\theta}{2} \right), \\ D_2 &= 16 \sin^2 \left(\alpha - \frac{\Delta\theta}{2} \right) \sin^2 \left(\alpha + \frac{\Delta\theta}{2} \right), \\ D_3 &= 16 \sin^2 \left(\beta - \frac{\Delta\theta}{2} \right) \sin^2 \left(\beta + \frac{\Delta\theta}{2} \right), \\ D_4 &= 4 \sin^2 \left(\frac{\Delta\theta}{2} \right). \end{aligned} \tag{10}$$

Requiring GOOFy invariance of the 2HDM potential one finds that only four options survive that correspond to different choices of $\Delta\theta \equiv \theta_1 - \theta_2$ and different relations between potential parameters, i.e.:

$$\Delta\theta = 0: \quad m_{11}^2 + m_{22}^2 = 0, \quad m_{12}^2 = 0, \quad \lambda_2 = \lambda_1, \quad \lambda_7 = \lambda_6,$$

$$\Delta\theta = \pi: \quad m_{11}^2 + m_{22}^2 = 0, \quad \lambda_2 = \lambda_1, \quad \lambda_7 = -\lambda_6,$$

$$\Delta\theta = \frac{2\pi}{3}: \quad m_{11}^2 + m_{22}^2 = 0, \quad m_{12}^2 = 0, \quad \lambda_2 = \lambda_1, \quad \lambda_5 = \lambda_6 = \lambda_7 = 0,$$

$$\Delta\theta = \frac{4\pi}{3}: \quad m_{11}^2 + m_{22}^2 = 0, \quad m_{12}^2 = 0, \quad \lambda_2 = \lambda_1, \quad \lambda_5 = \lambda_6 = \lambda_7 = 0.$$

GOOFy-invariant 2HDM

The following solutions allow for **all quark masses to be non-zero**.

Solution A: ($D_1 = D_2 = D_3 = D_4 = 0$), $\Delta\theta = \alpha = \beta = 0$,

$$\Gamma_1 = \begin{pmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \end{pmatrix}, \quad \Gamma_2 = -ie^{-i\theta_1} \begin{pmatrix} z_{11}^* & z_{12}^* & z_{13}^* \\ z_{21}^* & z_{22}^* & z_{23}^* \\ z_{31}^* & z_{32}^* & z_{33}^* \end{pmatrix}$$

Solution D-1: ($D_2 \neq 0, D_3 \neq 0, D_1 = D_4 = 0$), $\Delta\theta = 0, \beta = \alpha = \frac{\pi}{2}$,

$$\Gamma_1 = \begin{pmatrix} z_{11} & z_{12} & 0 \\ z_{21} & z_{22} & 0 \\ 0 & 0 & z_{33} \end{pmatrix}, \quad \Gamma_2 = -ie^{-i\theta_1} \begin{pmatrix} z_{22}^* & -z_{21}^* & 0 \\ -z_{12}^* & z_{11}^* & 0 \\ 0 & 0 & z_{33}^* \end{pmatrix}$$

Solution D-2: ($D_2 \neq 0, D_3 \neq 0, D_1 = D_4 = 0$), $\Delta\theta = 0, \beta = \alpha, \alpha \in (0, \frac{\pi}{2})$,

$$\Gamma_1 = \begin{pmatrix} z_{11} & z_{12} & 0 \\ -z_{12} & z_{11} & 0 \\ 0 & 0 & z_{33} \end{pmatrix}, \quad \Gamma_2 = -ie^{-i\theta_1} \begin{pmatrix} z_{11}^* & z_{12}^* & 0 \\ -z_{12}^* & z_{11}^* & 0 \\ 0 & 0 & z_{33}^* \end{pmatrix}$$

Relation set 1 ($\Delta\theta = 0$):

$$m_{11}^2 + m_{22}^2 = 0, m_{12}^2 = 0, \lambda_1 = \lambda_2, \lambda_7 = \lambda_6,$$

$$\Gamma_1 = -iX_\alpha^T \Gamma_2^* X_\beta^* e^{-i\theta_2}, \Gamma_2 = -iX_\alpha^T \Gamma_1^* X_\beta^* e^{-i\theta_1},$$

$$\Delta_1 = \pm iX_\alpha^T \Delta_2^* X_\gamma^* e^{i\theta_2}, \Delta_2 = \pm iX_\alpha^T \Delta_1^* X_\gamma^* e^{i\theta_1}.$$

- Using PyR@TE 3, we find RG-stability up to two-loop order for *Relation set 1* for Solutions A, D-1 and D-2 that allow for all quark masses to be non-zero.
- All solutions, A-M, have been confirmed RG stable up to two-loop order.
- Relations between potential parameters in Relation set 1, $m_{11}^2 + m_{22}^2 = 0, m_{12}^2 = 0, \lambda_1 = \lambda_2, \lambda_7 = \lambda_6$ (with $\Delta\theta = 0$) differ from the "original" relations for GOOFy/ r_0 symmetry (with $\Delta\theta = \pi$) of the scalar potential, $m_{11}^2 + m_{22}^2 = 0, \lambda_1 = \lambda_2, \lambda_7 = -\lambda_6$ that is part of Relation set 2 (which implies at least one massless quark).

Summary and conclusions

- A class of C-inspired GOUFy transformations has been defined and discussed.
- Bosonic version of GOUFy has been extended to fermions.
- In all cases considered, the constraints "implied" by GOUFy were RG stable at least up to 2 loops (adopting PyR@TE 3 code).
- Regardless if GOUFy is a genuine symmetry, resulting relations between parameters are RG-stable.
- Mass term in the SM is vanishing by GOUFy invariance.
- 2HDM might be consistent with GOUFy both in bosonic sector and in Yukawa interactions.
- In 2HDM a new all-order fixed point $m_{11}^2 + m_{22}^2 = 0$, $m_{12}^2 = 0$, $\lambda_1 = \lambda_2$, $\lambda_7 = \lambda_6$ (with $\Delta\theta = 0$) has been found in the scalar potential.

Summary and conclusions

- The constraints could be seen as emerging from the following "GOOFy/ r_0 symmetry":

$$x^\mu \rightarrow i x^\mu,$$

$$\Phi \rightarrow X_\phi \Phi^*, \quad \Phi^\dagger \rightarrow -\Phi^T X_\phi^\dagger, \quad A_\mu \rightarrow i A_\mu$$

$$\Psi \rightarrow -X_\psi \gamma_0 C \Psi^*, \quad \bar{\Psi} \rightarrow -\Psi^T C^{-1} i X_\psi^\dagger$$

- Application: finding RG stable relations between parameters.

Goofy is a cartoon character created by the Walt Disney Company.
He is a tall, anthropomorphic dog.

"Goofy" means silly, playful, ridiculous, or charmingly eccentric, often used to describe amusing behavior, or a person. It is generally a lighthearted term for foolishness.